Math 10C

Chapter 1 – Measurement

Program of studies and achievement indicators:

MATHEMATICS 10C

[C] Communication [CN] Connections [ME] Mental Mathematics and Estimation	[PS] Problem Solving [R] Reasoning [T] Technology [V] Visualization

Measurement	**General Outcome: Develop spatial sense and proportional reasoning.**
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators may be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve linear measurement, using: • SI and imperial units of measure • estimation strategies • measurement strategies. [ME, PS, V]	1.1 Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard and mile, and explain the choices. 1.2 Compare SI and imperial units, using referents. 1.3 Estimate a linear measure, using a referent, and explain the process used. 1.4 Justify the choice of units used for determining a measurement in a problem-solving context. 1.5 Solve problems that involve linear measure, using instruments such as rulers, calipers or tape measures. 1.6 Describe and explain a personal strategy used to determine a linear measurement; e.g., circumference of a bottle, length of a curve, perimeter of the base of an irregular 3-D object.
2. Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]	2.1 Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems. 2.2 Solve a problem that involves the conversion of units within or between SI and imperial systems. 2.3 Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion. 2.4 Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Measurement (continued)	**General Outcome:** Develop spatial sense and proportional reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators may be used to determine whether students have met the corresponding specific outcome.*
3. Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including: • right cones • right cylinders • right prisms • right pyramids • spheres. [CN, PS, R, V]	3.1 Sketch a diagram to represent a problem that involves surface area or volume. 3.2 Determine the surface area of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labelled diagram. 3.3 Determine the volume of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labelled diagram. 3.4 Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid or sphere, given the object's surface area or volume and the remaining dimensions. 3.5 Solve a problem that involves surface area or volume, given a diagram of a composite 3-D object. 3.6 Describe the relationship between the volumes of: • right cones and right cylinders with the same base and height • right pyramids and right prisms with the same base and height.
4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]	4.1 Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios. 4.2 Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle. 4.3 Solve right triangles. 4.4 Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean theorem. 4.5 Solve a problem that involves indirect and direct measurement, using the trigonometric ratios, the Pythagorean theorem and measurement instruments such as a clinometer or metre stick.

Chapter 1 (Topic 8) – Measurement

Day 1 - What is Measurement? Introduction to Metric (SI) and Imperial Units Section 1.1 & 1.2

*Start the class with the reading activity: **Encyclopedia Brown: The Case of the Bank Robbery** to help improve reading comprehension. This should take ~15 minutes.*

LESSON

Discuss with students different systems of measurement. What are measurements useful for? What jobs need measurement skills? What are some different units of measurement? What system of measurement do we typically use in Canada? What system of measurement do they use in America?

SI measuring system

– "syteme international d'unites" or **metric** – for example meters, centimeters etc.

-The base unit for measuring **length** is **the meter (m)**, the base unit for measuring **mass** is the **gram (g)** and the base unit for measuring **volume** is the **litre (L).**

-This system is a **DECIMAL** system because it is based on **multiples of 10**. Any measurement stated in one SI unit can be converted to another SI unit by multiplying or dividing by a multiple of 10.

Imperial system

– inches, feet etc. and is most commonly used in the United States and in many trades in Canada.

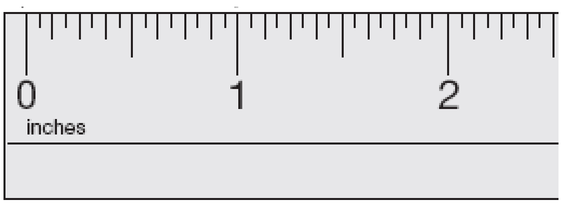
-This system is NOT a decimal system. This system is based on **referents.**

- A **referent** is used to estimate a measure; for example, the length from the tip of your thumb to the knuckle is ~ 1 inch.

This measurement system started in Ancient Roman Times. The current Emperor's foot length would be the standard unit for measuring length (distances). The length of a foot was later standardized to equal 12 inches.

-A fraction of an imperial measure of length is usually written in fraction form, not decimal form.

Consider the following:



This ruler has 16 divisions between inches, so the smallest indicated unit is $\frac{1}{16}$ of an inch.

-The following is an example where measurements with fractions from the imperial system can become quite tricky.

Show the following youtube video: American Choppers vs The Metric System

http://www.youtube.com/watch?v=Omh8Ito-05M

or search for "American Chopper Metric"

You may want to discuss with the students the issue; the bearded man was having difficulty properly subtracting his mixed number fractions

.

Converting Between Metric Units

We can easily convert between Metric units by multiplying or dividing by 10 each place holder that we move. A helpful way to remember the order of the SI units….

K H D m d c m

King **H**enry **D**anced **m**errily **d**own **c**ountry **m**eadows

Kilo Hecto Deca *m, g or L* deci centi mili

BASE UNIT

The base unit tells us what we are measuring, length (m), mass (g) or volume (L).

Ex. Briana wants to hang curtains in her kitchen. She measures the length of the window to be 250 cm. At the store, the curtain package lists the length in meters. How many meters of curtain will she need?

1) Place your finger on the unit you are starting with: **K H D m d c m**

2) Move your finger to the unit you want. Count how many times you have to multiply/divide by 10. **K H D m d c m**

← 2 spaces

4) Multiply/divide your current unit by 10 until you reach your desired unit.

We **HAVE** centimeters (cm). We **WANT** meters (m – base unit!)

We have to move **two** spaces to **the left** to get to our desired unit, so we have to **divide** by 10 **two times** to get the correct answer.

250 cm ÷ 10 ÷ 10 = 2.5 m

Briana will need to buy curtains with a length of 2.5 m.

Discuss with students other ways we could have gotten the same answer. Maybe we could have divided 250 by 100 (the same as dividing by 10 twice). Or we could have moved the decimal 2 places to the left.

Ex 2. Margaret is cleaning out her attic and has discovered a box of gold jewelry. She decides to sell it. She determines that she has 0.45 kg of gold. The current price of gold is $8.94 /g. How much money will she make?

1) First we need to change the mass from kg to g

2) We **HAVE** kilograms (kg). We **WANT** grams (g – the base unit!)

3) Start at kilo. We move three places to the **RIGHT** to get to gram. We have to multiply by 10 **three** times to get our desired unit.

0.45 x 10 x 10 x 10 = 450 g

OR 0.45 x 1000 = 450 g

Discuss other ways we could have determined this answer. Such as multiplying by 1000 or moving the decimal 3 places to the right.

4) Now we can multiply by the price of gold to determine how much money she will make: 450 g x $8.94 = $4 023 → wow! ☺

Discuss some common referents for metric units with students. For example, 1 cm = ~ the length of a fingernail, 1 m = ~ length of on arm from fingers to shoulder

Converting Between Imperial Units

- When measuring length with the imperial system, the following units are used:

Imperial Unit	**Relationship Between Units**	**Referent**
Inch (in. or “)		Thumb length to knuckle
Foot (ft or ‘)	**12 inches = 1 foot**	Foot length
Yard (yd)	**3 feet = 1 yard**	Arm span

Mile (mi) **1760 yards = 1 mile**

Ex. Russell wants to determine how far away the cafeteria is from Mrs. Lin's classroom in miles. He counts 1584 feet from the classroom to the cafeteria doors. How far is it in miles?

We HAVE **feet** we WANT **miles**

1 mile = 1760 yards , 3 feet = 1 yard. Use **unit analysis** to convert between units.

1584 feet $x \frac{1yd}{3ft} x \frac{1mi}{1760yd}$ = **0.3 miles**

*Show students that the units cancel out *unit analysis**

- Sometimes measurements will include feet and inches. If we want to convert to one unit only (either feet or inches) we must change one of the measurements to the desired unit.

Ex. Tristan is 5'11" tall. How tall is Tristan in inches?

We know that 5' = 5 feet and 11" = 11 inches. We WANT **inches** so we must convert 5 feet into inches first.

5 feet x 12 $\frac{inches}{foot}$ = **60 inches**

We can now add the 11 inches to determine Tristan's height in inches.

60 inches + 11 inches = **71 inches total**

Extension – Fractions in imperial measurements.

Ex. LeBron James (a basketball player for the Miami Heat) is 80 inches tall. Convert his height to **Feet and inches**

80 in x $\frac{1 ft}{12 in}$ = $\frac{80}{12}$ ft. Write this improper fraction as a mixed number.

- Review with students how to change improper fractions to mixed numbers:
  - How many times will 12 go into 80? **(6)**
  - What is the remainder? **(8)**
  - **Mixed number = 6** $\frac{8}{12}$

= 6 $\frac{8}{12}$ ft = 6 ft. 8 in.

note: Show students that $\frac{8}{12}$ ft x 12 in/ft = 8 in.

Extension – Map Scales

Ex. A map of Alberta has a scale of 1: 4 500 000. The distance on the map between Peace River and High Prairie is $4\frac{11}{16}$ in. What is the distance to the nearest mile?

- Review with students how to change a mixed number to an improper fraction:

$4\frac{11}{16}$ = 16 x 4 + 11 = $\frac{75}{16}$

- The map scale is 1 in represents 4 500 000 in.

$(\frac{75}{16})$ (4 500 000 in) = 21 093 750 in → now change this to miles.

21 093 750 in x $\frac{1ft}{12in} x \frac{1yd}{3ft} x \frac{1mi}{1760yd}$ **= 332.9190341**…. mi

- The distance between Peace River and High Prairie is approximately 333 miles.

Assignment

M10C Ch1 Metric Conversions W.docx

As well as: **1.1 Page 11 -12** # 3, 5, 7, 10, 11, 13, 14, 16, 17

Day 2 – Converting Between Metric and Imperial Section 1.3

*Start the class with the following activity. **Paper Folds to the Moon.docx This should take approximately 20 minutes. *Note: The document is a template of information for the teacher. It is not a hand out.***

LESSON

- Sometimes we may need to convert from SI units to Imperial, or from Imperial to SI units. To do this, we will require a chart that describes the basic conversion factors:

<table>
<tr><th colspan="4">Conversion Factors</th></tr>
<tr><th colspan="2">Length</th><th colspan="2">Capacity/ Volume</th></tr>
<tr><td>Imperial → Metric</td><td>Metric → Imperial</td><td>Imperial → Metric</td><td>Metric → Imperial</td></tr>
<tr><td>1 in $\doteq$ 2.5 cm</td><td>1 cm $\doteq \frac{4}{10}$ in
(1 cm $\doteq$ 0.4 in)</td><td>1 gal $\doteq$ 4.5 L</td><td>1 L $\doteq$ 0.22 gal</td></tr>
<tr><td>1 ft $\doteq$ 0.30 m
1 ft $\doteq$ 30 cm</td><td>1 mm $\doteq \frac{4}{100}$ in
(1 mm $\doteq$ 0.04 in)</td><th colspan="2">Mass/Weight</th></tr>
<tr><td>1 yd $\doteq$ 0.9 m
1 yd $\doteq$ 90 cm</td><td>1 m $\doteq 3\frac{1}{4}$ ft
(1 m $\doteq$ 3.25 ft)
1 m $\doteq$ 39 in</td><td>Imperial → Metric</td><td>Metric → Imperial</td></tr>
<tr><td>1 mi $\doteq$ 1.6 km</td><td>1 km $\doteq \frac{6}{10}$ mi
(1 km $\doteq$ 0.6 mi)</td><td>1 oz $\doteq$ 28.35 g</td><td>1 g $\doteq$ 0.0305 oz</td></tr>
<tr><td colspan="2" rowspan="2">*note: 1 m $\doteq$ 1 yd</td><td>1 lb $\doteq$ 0.454 kg</td><td>1 kg $\doteq$ 2.205 lb</td></tr>
<tr><td>1 lb $\doteq$ 454 g</td><td>1 g $\doteq$ 0.002 lb</td></tr>
</table>

**note: These conversions are approximate. oz = ounce, lb = pound, gal = gallon*

To convert between units, we will use proportions:

1) Set up a fraction : $\frac{WANT}{HAVE}$ Place an "x" and the units for what you WANT.

2) Find the appropriate conversion on the chart.

3) Set up a second fraction, with the units of what you WANT in the numerator and what you HAVE in the denominator

4) Perform opposite operations to solve for your unknown.

Ex1. Carlos measures 14 cm but he needs to know this length in inches. How many inches is 14 mm?

14mm = _______in ?

- WANT **inches** HAVE **centimeters**
  $\frac{x\_in}{14cm}$

- we are converting from <u>Metric → imperial</u>, so we will use: 1 cm $\doteq$ $\frac{4}{10}$ in $\doteq$ 0.4 in

- $\frac{0.4in}{1cm}$

- $\frac{x\_in}{14cm} \doteq \frac{0.4in}{1cm}$

Multiply both sides by 14 cm $x\_in \doteq 14cm\left(\frac{0.4in}{1cm}\right) \doteq 5.6$ in, or $5\frac{3}{5}in$

Discuss how we changed 5.6 to 5 and 3/5 (we have 5 whole inches, and 0.6 an inch, 0.6 = 3/5

Ex2. You are on vacation in Hawaii and are going on a mountain hike. You read in the guide book that the hike is 3.5 miles. How many km is this?

- WANT **km** HAVE **mi**
- $\frac{x\_km}{3.5mi}$
- we are converting *Imperial* → *metric* so we will use: 1 mi = 1.6 km
- $\frac{1.6km}{1mi}$
- $\frac{x\_km}{3.5mi} \doteq \frac{1.6km}{1mi}$

Multiply both sides by 3.5 mi $3.5mi\left(\frac{1.6km}{1mi}\right) \doteq$ **5.6 km**

- We may need to complete more than one conversion if what we want is not on the conversion factor chart.
- For example, you may need to change from Imperial →SI first , then convert the SI units to the desired unit.

Ex3. At least once a year, a truck will get stuck on the High Level Bridge in Edmonton. The bridge has a low clearance of 10’6”. A truck driver knows that her semitrailer is 3.3 m high. Will her vehicle fit under the bridge? Or will she be stopping traffic?

- WANT **Feet + Inches** HAVE **Meters**

$\frac{x\_ft}{3.3m}$

- We are converting *Metric* → *Imperial* so we will use 1 m $\doteq 3\frac{1}{4}$ ft (1 m $\doteq$ 3.25 ft)
- $\frac{3.25ft}{1m}$
- $\frac{x\_ft}{3.3m} \doteq \frac{3.25ft}{1m}$

Multiply both sides by 3.3 m $3.3m\left(\frac{3.25ft}{1m}\right) \doteq$ 10.725 ft or 10 $\frac{29}{40}$ ft

The trailer is approximately 10 $\frac{29}{40}$ ft or ~10 ft. 9 in, high; so it is too tall to fit under the bridge!

Show students how to change $\frac{29}{40}$ ft *to inches (*$\frac{29}{40}$ *x 12 in/ft = ~ 9 in.) How could the truck driver get the trailer to fit? Release some air from the tires!*

This height is an estimate though. To make sure the vehicle will not fit, we will calculate an exact conversion. We will use 2.54 cm = 1 in.

3.3 m = 330 cm

$\frac{xin}{330cm} = \frac{1in}{2.54cm}$

Multiply both sides by 330 cm $= 330cm\left(\frac{1in}{2.54cm}\right) = 129.92125\ldots$in

Now, convert inches to feet **remember, do not change your answer until the last step!*

129.92125….in x $\frac{1ft}{12in}$ = 10.825771….ft. or

This measurement is a little MORE than 10 ft. 9 in, so it is DEFINITELY too large to fit underneath the bridge!

Assignment:

Activity – Golden Ratio (discussing the golden ratio of the perfect face – ruler measurements and ratio calculation activity).

AND: **1.3 Page 22-23** #4, 5, 6, 9, 10, 11, 15, 17

Day 3 - SA right pyramids and cones Section 1.4

Start this class with a review of conversions between metric and Imperial Units. Write the following example on the board for the students to calculate.

1 lb ≐ 0.454 kg	1 kg ≐ 2.205 lb

*note: lb = pound

22 300 lbs is how many kg? **(Answer: ~10 124.2 kg)**

Show the video "Gimli Glider" on youtube.com – a story about an Air Canada jet that had to crash land in Gimli, Manitoba because they ran out of fuel.

The video can be found here: http://www.youtube.com/watch?v=hl8foT-v6Vg

Or you can search for "Gimli Classroom Version" by SalMathGuy. There are two parts, totaling ~20 minutes.

*After the video, discuss with the class the implications of properly converting Imperial and Metric units. The fuel technicians did not do the proper conversion, and as such, they only fueled the jet with half as much fuel as it would need (the conversion that the students did before the video). They actually needed 22 300 kg of fuel. How many pounds of fuel is this? (~**49 171.5 lbs)** which is twice as much as the plane was given!*

LESSON - hand out "*M10C basic shapes review.docx*" to students and read through

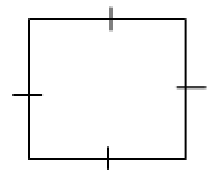
- People who work in the trades, such as carpenters, plumbers and electricians often use measurements to solve problems. There are usually many 2-D and 3-D shapes involved in measurements.

- **2-D** means "two-dimensional"; These shapes are flat, and you can draw them on a piece of paper. When we work with 2-D shapes we are usually considering **perimeter and/or area.** 2-D drawings are used for floor plans of buildings, yards, parks, etc.

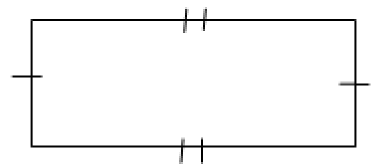
- **3-D** means “three-dimensional”; These shapes have depth, and are difficult to draw on a piece of paper. A three dimensional shapes would include a box, soup can, ball, most of the objects that we use in our everyday lives. When we work with 3-D shapes we are usually considering **surface area and/or volume.**

- <u>**Basic Shapes (REVIEW)**</u>

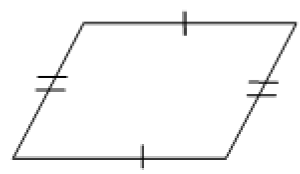
- **Polygon –** a closed shape that consists of line segments.



- **Quadrilateral - ** a shape with four sides, such as a **square, rectangle, parallelogram, or trapezoid.**
- **Square** - all sides are of an equal length
- **Rectangle** – two sides are the same length, and the other two are the same length. We usually call this **width** and **length**

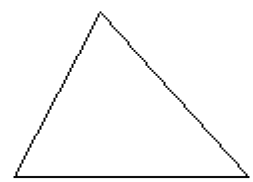


- **Parallelogram** – usually described as a “slanted” rectangle. **Parallel** means that two (or more) lines will never cross. Squares and rectangles are technically parallelograms.



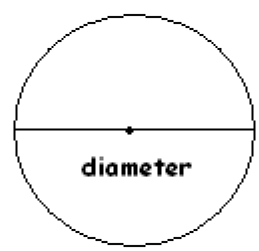
- **Trapezoid - ** a four sided shape with only one set of parallel lines.

- **Triangle –** Three sided figure.

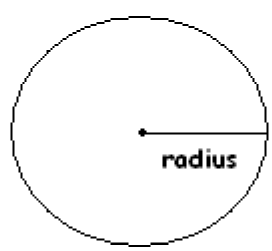


- **Circle -** All the points on a circle are equidistant (the same distance) from the CENTER of the circle.

- A line that passes through the center of a circle and touches the edge of the circle on both sides is called the diameter.



- A line that starts at the center of the circle and touches an outside edge is called the radius.



- The **circumference** of a circle is the perimeter of the circle. It can be calculated with the formula : $C = \pi d$

  Where C = circumference (perimeter)

  π = "pi" a constant that is 3.14….

  d = diameter

- **Perimeter** – the distance around the outside of an object. You can calculate the perimeter of a shape by adding up the lengths of each side.

- **Area** is the surface of a 2-D object. A good way to visualize area is to imagine entirely coloring in between the lines of a shape; when you have done this, you have colored the area.

- When we calculate the area, we are finding the "square" units of the shape.

- **Area Square** = side x side = s x s OR $\mathbf{s^2}$
- **Area Rectangle** = length x width = **l x w**

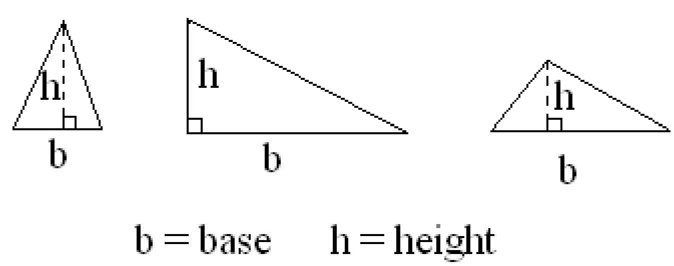
- The **height** of an object is the perpendicular distance from the base of a polygon to an opposite vertex.

- **Perpendicular** – two lines that form a right (90^0) angle. For example, a corner is a 90^0 angle.

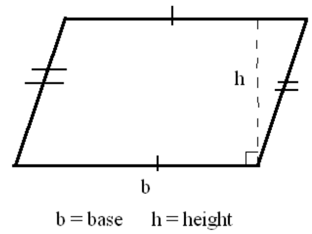
- **Vertex** – the point where two or more lines meet.

- **Area triangle** = $\frac{1}{2}$ (base x height) OR

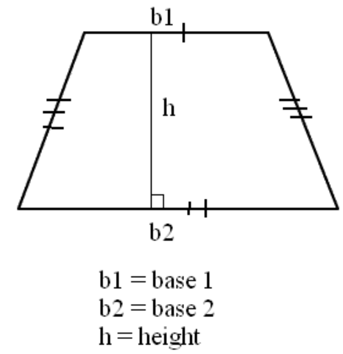
  $\frac{1}{2}$ **b x h**



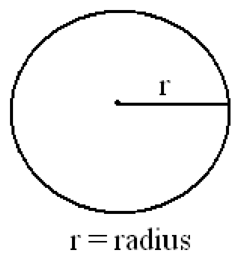
- **Area Parallelogram =** base x height OR **b x h**



- **Area Trapezoid =** $\frac{1}{2}$ (base + opposite side) x height OR $\frac{1}{2}$ **(b_1+b_2) x h**



- **Area Circle =** π x radius x radius OR πr^2



<u>LESSON – Surface Area of Right Pyramids and Right Cones</u>

- Most of the solid objects we are familiar with are 3-D or "3-Dimensional". This means that these objects have length, width and height/depth.

- A **<u>right pyramid</u>** is an object that has triangular faces and a base that is a polygon. The shape of the base determines that name of the pyramid.

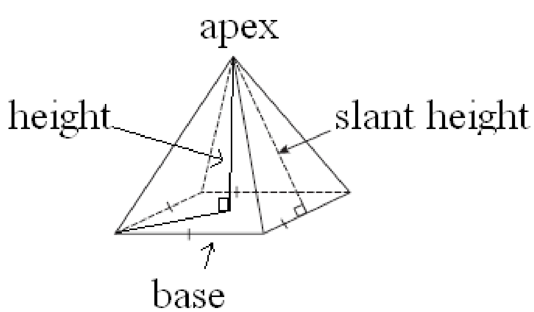
Each side of a prism/pyramid is called a **<u>face</u>** – a 2-D object that forms a flat surface of a 3-D object.

- The **surface area** of a 3-D object is the **<u>sum</u>** of all the **<u>areas</u>** of the faces of the object.
- Surface area can be easy to calculate if you are able to visualize the 2-D shapes (faces) that make up the 3-D object.

- Today we will be focusing on the surface area (SA) of right pyramids and cones.

- The triangular faces of a right pyramid meet at a point called the **apex**. The *height* of the pyramid is the perpendicular distance from the apex to the center of the base.

- When the base of a right pyramid is a regular polygon, the triangular faces are congruent (**the same**). The **slant height** of the right pyramid is the height of the triangular face.

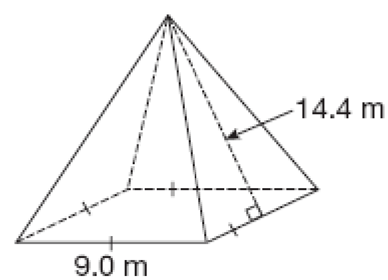


- Imagine the **net** of the right square pyramid above. A **net** is the flattened out version (2D) version of a 3D object. *Have a set of geoblocks handy to show the students a net of a pyramid.*

- How many faces are there? **5**

- What shapes do you see? **4 triangles, 1 square**

- Ex1. Here are the dimensions of the pyramid. Calculate the surface area.



  - Area of triangular face **=** $\mathbf{A_{Triangle}}$ **= ½ bh = ½ (9.0m)(14.4m) = 64.8 m^2**
  - Area of square base **=** $\mathbf{A_{Square}}$ **= s^2 = 9.0m^2 = 81 m^2**
  - **The SA of the pyramid is:**
  - SA = 4(64.8m^2) + (81 m^2) = **340.2m^2**

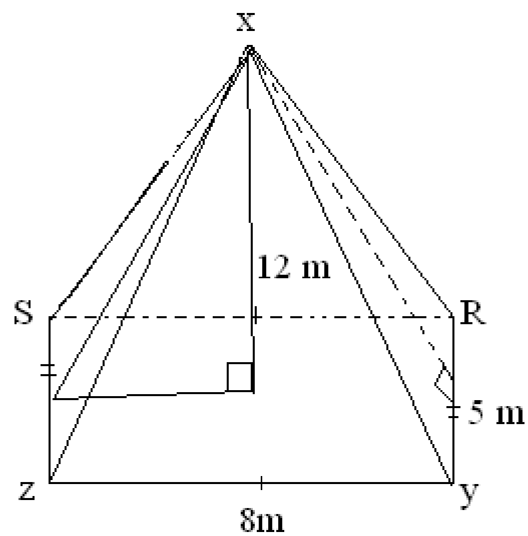
Explain why we multiplied 64.8m^2 by 4

***NOTE:** The **lateral area** is the area of the triangular faces of a pyramid.

- Sometimes, what we learned in our trigonometry unit can be useful to help calculate the slant height of a right pyramid. Remember Pythagoras? $\mathbf{a^2 + b^2 = c^2}$

- Ex2. A right rectangular pyramid has base dimensions 5 m by 8 m and a height of 12 m. Calculate the surface area of the pyramid to the nearest square meter.

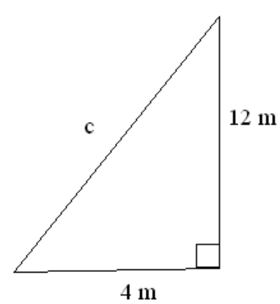
Sketch it out!



- Since this is a **rectangular pyramid**, there are two sets of congruent triangles; one set has a base of 8 m (ΔXYZ and ΔXSR), one set has a base of 5 m (ΔXRY and ΔXSZ)
- We can find slant height of each face using Pythagoras.

- We know that the height of the pyramid is from the **center** of the base; therefore, in the diagram above, to find the slant height of the triangle face with a base of 5 m, we will use:

    - *Explain to students how we calculated 4m*

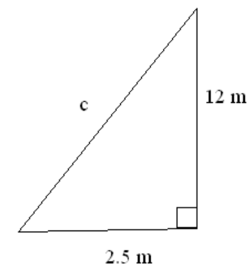


c = $\sqrt{4^2 + 12^2} = \sqrt{160}$ m

The area of ΔXRY and ΔXSZ can now be calculated:

A= ½ bh = ½ (5) ($\sqrt{160}$) = $2.5\sqrt{160}$ m^2

- Now, calculate the slant height of the triangle with a base of 8 m:
    - *Explain to students how we calculated 2.5 m*



$c = \sqrt{2.5^2 + 12^2} = \sqrt{150.25}$

The area of ΔXYZ and ΔXSR can now be calculated:

$A = ½\, bh = ½\, (8)\, (\sqrt{150.25}\,) = 4\sqrt{150.25}\ m^2$

Now, calculate the area of the base of the pyramid:

$A_{base} = l \times w = 8 \times 5 =$ **40 m²**

Now, add together to get the total Surface Area of the pyramid.

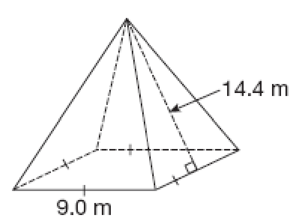
$SA = 2\,\mathbf{(2.5)(\sqrt{160}\,)} + 2\,(4)(\sqrt{150.25}\,) + 40 = 201.306\ldots m^2 =$ **201 m²**

- We can use the formula **SA = ½ s (perimeter of base) + (base area)**
    - Where s = slant height

  to calculate the surface are of any right pyramid with a regular polygon base. Read page 30 in your text book which describes how this formula is derived.

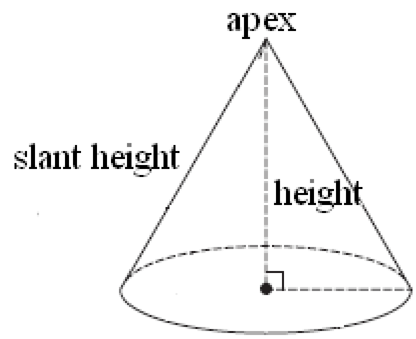
Let's try this formula with the first example:

SA = ½ (14.4)(4(9.0)) + (9.0²) = 340.2 m² Same answer! Neat!



**explain to students why we used 4(9.0)*

- A right circular cone is a 3-D object that has a circular base and a curved surface. The ***height*** of the cone is the perpendicular distance from the apex to the base.

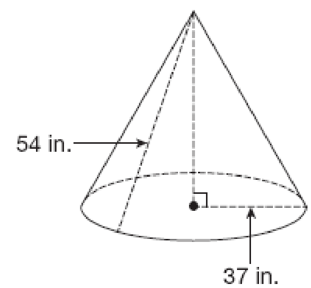


- The formula for the area of a right cone is $SA = \pi rs + \pi r^2$

see page 31/32 of your text book to read about how this formula is derived.

→*Which part is representing the area of the circle?* πr^2

→*Which part is representing the lateral area?* πrs



Ex3. Calculate the area of the following right cone to the nearest inch:

$SA = \pi rs + \pi r^2$ = $\pi(37)(54)$ + $\pi(37)^2$ = 10 577.74…in^2 = **10 578 in^2**

Extension – Lateral Area

Ex4. The later area of a cone is 416 cm^2. The diameter of the cone is 18 cm. Determine the height of the cone to the nearest tenth of a centimeter.

Sketch it out! First, solve for the slant height, s.

Find the radius! = **9 cm**

$A_{lateral} = \pi rs$

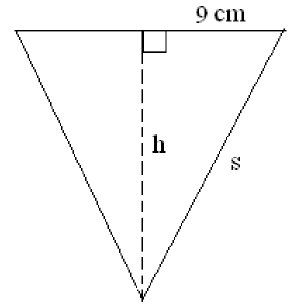
416 = $\pi(9)$s

Divide both sides by 9π

$$\frac{416}{9\pi} = \frac{9\pi s}{9\pi}$$

s = $\frac{416}{9\pi}$

s = 14.712…..



To determine the height of the cone, use the Pythagorean Theorem:

$9^2 + h^2 = s^2$

Substitute for s

$81 + h^2 = (14.712….)^2$

Solve for h

h = $\sqrt{(14.712...^2 - 81}$

h = 11.6392…. = **11.6 cm**

Assignment:

1.4 Page 34 – 35 #5, 7, 8, 9, 10, 11, 13, 15, 16, 20, 21

Day 4 – SA of other shapes

Start the class with the reading activity: ***Encyclopedia Brown: The Case of Merko's Grandson*** *to help improve reading comprehension. This should take ~10 minutes.*

You will need a set of geoblocks for this lesson, to help students visualize 3D prisms and nets.

LESSON

- Today we will focus on calculating the surface area of other shapes, prisms and cylinders.

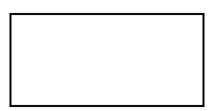
- A **prism** is a 3-D shape whose "bases" (or ends) are of the same size and shape and are parallel to one another. The base shape of a prism is usually described in the name. For example, a triangular prism has a base shape of a triangle.

- A **cylinder** is a 3-D shape that has circles for the base. The most common cylinder is the soup can!

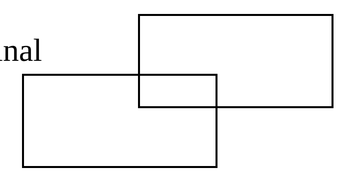
Practice Drawing 3-D shapes

Let's practice drawing a **rectangular prism**

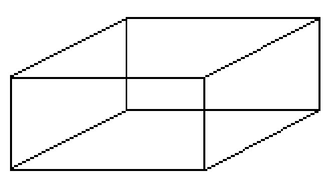
Step 1: Draw the base shape (rectangle)



Step 2: Draw the exact same rectangle, above and slightly to the right of the original



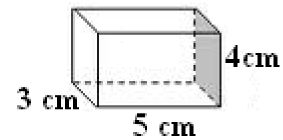
Step 3: Connect the corners of your first rectangle and your second rectangle



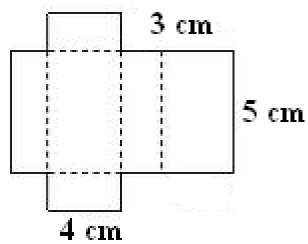
- Just like when we calculate the surface area of right cylinders and cones, we need to determine the total area of each face that make up the 3D object. Sometimes it is easiest to visualize the net of the object. Once you have determined the area of each face, add these together to get the total SA.

- Ex1. Determine the surface area of the following rectangular prism:

  Show students an example of a rectangular prism using the geoblocks.



- The net would look like this:



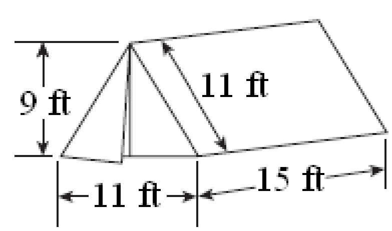
- Notice how there are two sides that are 4 cm x 3cm, two sides that are 4 cm x 5 cm and two sides that are 3 cm x 5 cm? We can write a formula to describe the SA of a rectangular prism:
  - *SA = 2(l x w) + 2(l x h) + 2(w x h)*
    - Where *l* = length

      w = width

      h = height

- SA = 2 (5 x 3) + 2(5 x 4) + 2(3 x 4) = **94 cm^2**

- **Ex2. A)** Determine the surface area of the following tent:
  - What is the name of this prism? *Triangular prism*
  - How many faces? *Five*



Visualize the shapes that make up this prism:

Two triangles, b = 11 ft, h = 9ft

Three rectangles, l = 15 ft, w = 11 ft

Calculate the SA.

SA = 2(½ bh) + 3(lw) = 2(½ 9 x 11) + 3(15 x 11) = **594 ft^2**

B) If canvas costs $0.40 per square foot, how much will the fabric cost to make the tent?

$0.40/$ft^2$ x 594 ft^2 = **$237.60**

C) What is the surface area of the exposed part of the tent?

Explain to students how the bottom of the tent is "hidden" on the ground.

SA = 2(½ 11 x 9) + 2(15 x 11) = **429 ft^2**

OR

Hidden part Area = 15 x 11 = 165 ft^2

Total SA = 594 ft^2

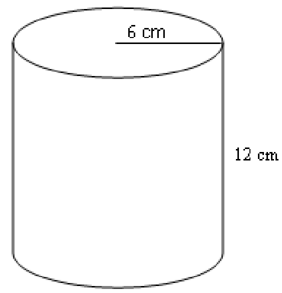
594 ft^2 – 165 ft^2 = **429 ft^2**

*Handout **Finding the Cylinder Formula.docx** to students. You will need approximately 10-15 cans of soup for this activity, pieces of string for each student group, and rulers. Students can work in groups of 2-3 for this activity. Donate the soup to the food bank after, or save for the next semester!*

Extension questions (after the cylinder activity)

A cylinder has a height of 12 cm and a radius of 6 cm. Determine the surface area to the nearest 10^{th} of a cm^2

Sketch it!



$SA = 2\pi rh + 2\pi r^2$

SA = $2\pi(6\text{x}12) + 2\pi(6^2)$ = **678.6cm^2**

EXTENSION ACTIVITY

Write the following question on the board:

A circle has a diameter of 8". A new circle is created to meet one of the conditions below. What is the diameter of the new circle?

Option 1: The new circumference is 5 times as big

Option 2: The new area is ¼ as big

Questions to discuss afterwards:

Is the new diameter likely to be greater than or less than the original?

Is the circumference of the new circle going to be greater than or less than the original one? What about the area?

Write the following on the board:

Two circles have different diameters.

Option 1: Can they have the same area?

Option 2: Can they have the same circumference?

Questions to discuss afterwards:

Did you try an example?

How many examples would you have to try to be sure?

What measurement formula did you use to help you?

Why did you use that formula?

How can using the formula help you answer the question?

Assignment: M10C SA other shapes.docx

Day 5 – Volume of Right Pyramids and Right Cones Section 1.5

Start this lesson with the reading "The World's Largest Hamburger", handout

M10C Ch1 Hamburger.docx *Tell your students they have 8 minutes to read the story and to come up with 5 questions about the story. After 8 minutes, ask the students what their questions are; see how many are similar. Encourage silly questions (for example: "how many cows does it take to make that much hamburger?"). Pick one of their questions, and use the internet to investigate!*

Here are some sample follow up questions:

1) **What is the area of the top of this hamburger?**
2) **How many cows would it take to make this hamburger? *note average cow ~568 lbs**
3) **How long would it take to cook this hamburger?**
4) **How many students would it take to make this hamburger?**
5) **How much ketchup would you need?**
6) **How much hamburger did each person get?**

LESSON

- **Volume** is the amount of space occupied by a 3-D object.

- We are most familiar with expressing volume as liters (L), milliliters (mL), gallons (gal), pints (pt), even cups when baking!

- *Did you know that 1 cup is approximately 240 mL? Then 2 cups must be about 500 mL -- half a litre!*

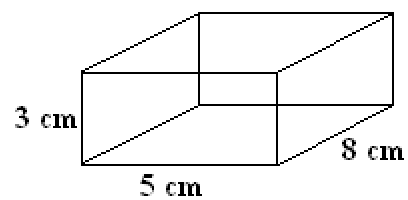
- It can be difficult to calculate the volume of 3-D objects in L or mL. These are called **capacity units.** For this course, we will calculate volume in Imperial Units^3 or Metric Units^3.

- The **units** for volume are said to be "cubed" or to the power of 3. For example, cm^3 or m^3.

- To calculate the volume of a rectangular prism we use the formula….

V = *lwh* Where l = length w = width h = height

Ex1. Calculate the volume of the following rectangular prism:



Volume = l x w x h = 8 cm x 5 cm x 3 cm

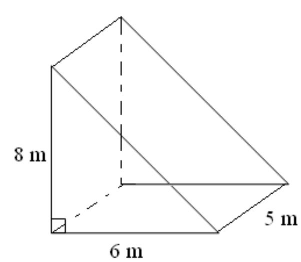
= **120 cm^3**

- To calculate the volume of other prisms, we use the formula :

Volume = *Ah* where A = base area h = height

Ex.2 Calculate the volume of the following triangular prism :

**note : The base shape is a triangle, the height is the part that connects the two base shapes.*



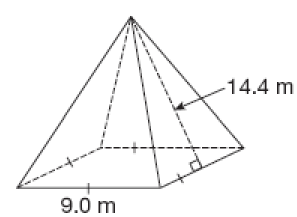
Volume = *Ah* = (½ x 6 x 8) (5) = **120 m^3**

Using geoblocks, select a right prism and a right pyramid with an equal height and base. Fill the pyramid with sand. Pour the sand into the right prism. See how many pyramids are required to fill the prism. It should be 3 pyramids.

- The volume of a **right pyramid** can be calculated using the following formula :

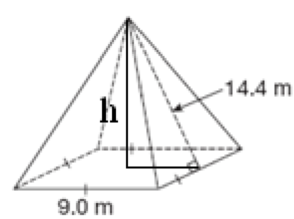
$V = \frac{1}{3}Ah$ Where A = area of the base h = height

Ex3. What is the volume of the following right square pyramid to the nearest cubic meter?



*First, we need to calculate the HEIGHT of this pyramid.

Use Pythagoras theorem!



$4.5^2 + h^2 = 14.4^2$

h = $\sqrt{14.4^2 - 4.5^2}$ = $\sqrt{187.11}$

Now, determine the volume:

$V = \frac{1}{3}Ah = \frac{1}{3}(9.0^2)(\sqrt{187.11}) = 369.328\ldots =$ **$369\ m^3$**

Use geoblocks and select a right cylinder and a right cone with the same height and base. Determine how many cones of sand are required to fill the cylinder. (3)

- To calculate the volume of a cylinder we use the formula:

$V = \pi r^2 h$ where r = radius and h = height

- To calculate the volume of a cone we use the formula:

$V = \frac{1}{3}\pi r^2 h$ where r = radius and h = height

Ex4. The volume of a cylinder is 450 mm^3. If the radius is 5 mm, what is the height to the nearest tenth of a mm?

$V = \pi r^2 h$

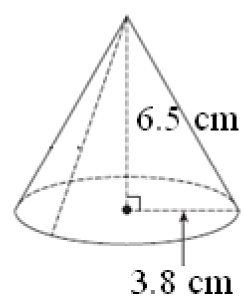
450 = $\pi(5^2)$h

Divide both sides by $\pi(5^2)$

$\frac{450}{\pi 5^2}$ = h

h = **5.7 mm**

Ex5. What is the volume to the nearest cubic cm of the following cone?



$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3.8^2)(6.5) = 98.289\ldots =$ **98 cm^3**

EXTENSION

Write the following on the board:

Two cones have the same base, but the second cone is 5 times as high as the first one.

Option 1: How are their volumes related? Is that always true?

Option 2: How are their surface areas related? Is that always true?

Have students pick option 1 or 2. After ~10 minutes, discuss the following questions:

What formula did you use to solve the problem?

What relationships do the measurements have? How did you figure that out?

Do the specific values of the radius and the height affect the relationship?

Assignment: 1.5 Pages 42-43 # 4, 5, 6, 7, 8, 9, 10 b, 11, 12, 14, 18, 19

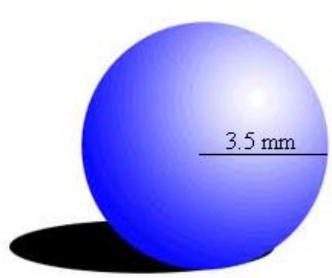
Day 6 - SA and Volume of a Sphere Section 1.6

*Start with the reading activity: "**Encyclopedia Brown and the Case of the Gym Bag**" This should take ~10 minutes.*

LESSON

*Handout **Finding the Sphere Formula.docx** to each student. You will need ~10-15 oranges (mandarin oranges will work), string, rulers, compass, and scrap paper for each student.*

Ex.1 Given the following sphere, calculate the surface area to the nearest tenth square mm:



$SA = 4\pi r^2$

SA = $4\pi(3.5^2)$ = 153.93…. = **$153.9mm^2$**

- The formula for the volume of a sphere is explained in your text book on page 48.
- $V = \frac{4}{3}\pi r^3$

Ex2. Calculate the volume of the sphere from example 1 to the nearest tenth of a cubic mm.

V = $\frac{4}{3}\pi(3.5^3)$ = 179.59… = **$179.6\ mm^3$**

Ex3. The volume of a sphere is 524 cm^3. What is **diameter** of the sphere?

$V = \frac{4}{3}\pi r^3$

524 = $\frac{4}{3}\pi r^3$

Multiply both sides by 3

3(524) = $4\pi r^3$

1572 = $4\pi r^3$

Divide both sides by 4π

$\frac{1572}{4\pi} = r^3$

125.09….. = r^3

Find the cube root of both sides to solve for radius

$\sqrt[3]{125.09....}$ = r

r = 5.0012… = 5.0 cm

Diameter =2r = 2(5) = **10 cm**

Ex4. A hemisphere is a sphere cut in half. What is the surface area of a hemisphere with a radius of 3.0 m to the nearest tenth of a square meter?

Sketch it!

First, calculate the SA of the sphere:

$SA = 4\pi r^2$ = 4 π (3^2) = <u>113.097….m^2</u>

Divide by two to get the hemisphere

113.097…./2 = <u>56.548…. m^2</u>

*This is NOT the area though! Consider, there is a CIRCLE on the top of the hemisphere. The area of this circle needs to be calculated as well!

A_{circle} = $\pi r^2 = \pi (3^2) = 28.27....m^2$

Add together to get the total SA of the hemisphere!

56.548…. + 28.27…. = 84.823… = **<u>84.8m^2</u>**

- To calculate the volume of a hemisphere, we use the formula:

  $$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

  ➔ $V = \frac{2}{3}\pi r^3$

Assignment: 1.6 Page 51-52 #3, 4, 5, 7, 9, 10, 11, 13, 15, 18, 19, 21, 23

Day 7 – SA and Volume of all Objects (Composite Solids) Section 1.7

Start with the reading activity "Encyclopedia Brown and the Case of Shoeless Sam". This should take ~10 minutes.

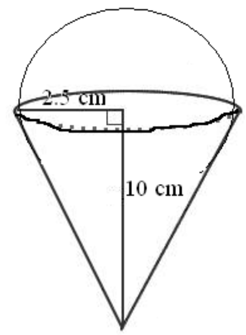
LESSON

- A composite solid is a 3-D object made up of two or more 3-D objects.
- To calculate the volume of a composite solid, determine the volume of each shape individually and then add together.
- Calculating the surface area of composite solids can be a little more difficult. You have to consider that some of the surfaces may be "hidden" within the object.

Ex1. An ice cream cone with a height of 10 cm and a radius of 2.5 cm is filled with ice cream. A hemisphere of ice cream (with the same radius) sits on top.

a) What is the volume of ice cream to the nearest tenth of a cubic centimetre?

Sketch it!



Volume of a hemisphere = $V = \frac{2}{3}\pi r^3$

Volume of a cone = $V = \frac{1}{3}\pi r^2 h$

$\sqrt{(2.5^2) + (10^2)}$

Total Volume = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi\,(2.5^3) + \frac{1}{3}\pi\,(2.5^2)(10)$

= 98.174…. = **98.2 cm^3**

b) What is the surface area to the nearest square cm of this composite solid?

*We need to consider the exposed parts only. We have ½ a sphere, and a cone with out the top circle exposed.

$SA_{sphere} = 4\pi r^2$ $SA_{cone} = \pi r^2 + \pi r s$

- We know that πr^2 from the cone formula represents that circle part. We also know that we have to divide SA of sphere by 2 to get a hemisphere:

$SA = 2\pi r^2 + \pi r s$

Explain to students which each part represents.

- We also need to calculate the slant height to complete this equation.

$a^2 + b^2 = c^2$

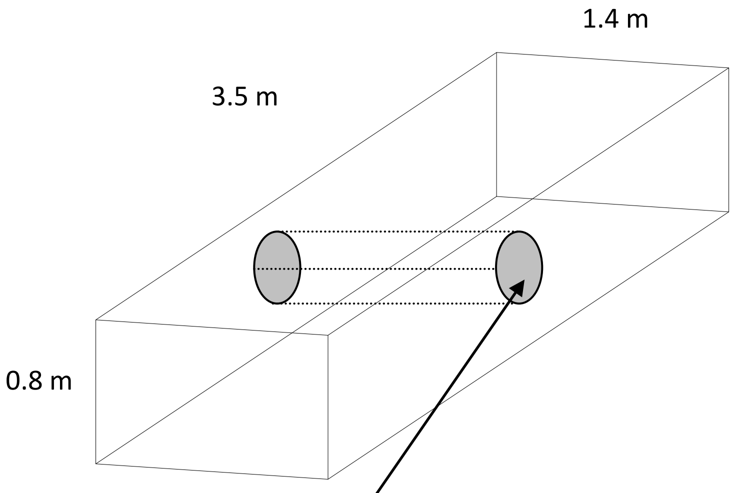
$c^2 = (2.5^2) + (10^2)$

$c = \sqrt{(2.5^2) + (10^2)} = \sqrt{106.25}$

- Now we can calculate the SA of the composite solid.

SA = $2\pi(2.5^2) + \pi(2.5)(\sqrt{106.25}) = 120.2268\ldots$ = **120.2 cm²**

Ex2. A stock watering tank is in the shape of a rectangular prism, with a cylinder heater installed as shown in the diagram. Calculate the volume of the tank to the nearest cubic meter when filled with water



First, we need to convert all of our units to the same unit. Convert 44 cm to m.

44 cm x $\frac{1m}{100cm}$ = 0.44m = diameter

Radius = 0.44/2 = 0.22 m

Now, let's consider the shapes we have in this composite solid:

A rectangular prism, and a cylinder.

$V_{prism} = lwh$ $\qquad V = \pi r^2 h$

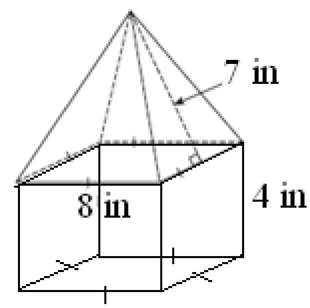
We need to **subtract** the volume of the cylinder from the volume of the prism.

V_{solid} = lwh - $\pi r^2 h$ = $[(3.5)(1.4)(0.8)] - [\pi(0.22^2)(1.4)] = 3.70\ldots$ = **3.7 m³**

Explain to students how we knew the height of the cylinder was the same as the width of the rectangular prism.

Ex3. Calculate the surface are of the following composite solid to the nearest square inch.

- Consider the sides that are "hidden". How many exposed faces are there? (9)
  You may want to get a set of geoblocks to show the students how the top of the prism is hidden, and the bottom of the pyramid is hidden.



$SA_{pyramid}$= ½ s (perimeter of base) + (base area) *SA_{prism} =2(lw)+2(lh)+2(wh)*

SA_{object} = ½ s (perimeter of base) + 1(lw)+4(lh)

= ½ (7)(8+8+8+8) + 1(8x8)+4(8 x 4) = **304 in^2**

Explain to students how we developed the formula. Note how the four sides of the prism are of equal length and height.

Extension

Ex4. A cylindrical water tank that is 5.2 m high with a radius of 3.1 m is filled with water. The water drips from the tank into a rectangular prism trough. If the volume of water is completely transferred from the cylinder to the trough, what is the height of the water (to the nearest m) if the length of the trough is 5 m and the width is 2 m?

First we need to determine the volume of the cylindrical water tank

$V = \pi r^2 h$

$\frac{156.991...}{10}$

V = π(3.1^2)(5.2) = 156.991….m^2

If this volume is completely transferred to the trough, we will assume the volumes will be the same.

$V_{trough} = lwh$

$156.991\ldots = (5)(2)(h)$

$156.991\ldots. = (10)(h)$

Divide both sides by 10

$$\frac{156.991...}{10} = h$$

h=15.699… = **16 m**

Assignment: 1.7 Page 59 -61 #3, 5, 6, 7, 8, 9, 10, 11, 13

Day 8 – Quiz

M10C Ch1 QuizA.docx

Reading Assignment: Wired Magazine Improper Conversion

Day 9 – Review

- *Go over quiz.*
- *Review*

M10C Ch1 Review.docx

Day 10 – Exam

M10C SAL Ch1 Unit ExamA.docx