Course:	Math 10C
Unit of Study:	Polynomial Products and Factors

Step 1: Identify the Outcomes to Address

Guiding Questions:

- What do I want my students to learn?
- What can they currently understand and do?
- What do I want my students to be able to understand and do based on the big ideas and specific outcomes in the program of studies?

Outcome(s) from the Program of Studies:

Algebra and Number
(S01– students must have complete the real numbers unit prior to this unit of study so that SO1 will be covered already)
SO1. Demonstrate an understanding of factors of whole numbers by determining the:

• prime factors
• greatest common factor
• least common multiple
• square root
• cube root.
[CN, ME, R]

SO4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.
[CN, R, V]

SO5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.
[C, CN, R, V]

Step 2: Learning Trajectory

Prior Learning	Current Learning	Future Learning
Grade 5 Number Strand 4. Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property. [C, CN, ME, R, V] **Grade 7** Patterns and Relations (variables and Equations) S.O. #4. Explain the difference between an expression and an equation. [C, CN] **Grade 9:** Patterns and Relations (variables and Equations) S.O. #5 Demonstrate an understanding of polynomials (limited to polynomials of a degree less than or equal to 2) SO. #6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). S.O. #7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V] **Grade 9** **Number Strand**	**Algebra and Number** **SO1.** Demonstrate an understanding of factors of whole numbers by determining the: • prime factors • greatest common factor • least common multiple • square root • cube root. [CN, ME, R] **SO4.** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V] **SO5.** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]	**20 –1** **Algebra and Number** S.O 4. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [C, ME, R] 5. Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, ME, R] 6. Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). [C, PS, R] **20–1 Relations and Functions** 1. Factor polynomial expressions of the form: • $ax^2+bx+c, a\neq 0$ • $a^2x^2-b^2y^2, a\neq 0, b\neq 0$ • $a(f(x))^2+b(f(x))+c, a\neq 0$ • $a^2(f(x))^2-b^2(g(y))^2, a\neq 0, b\neq 0$ where *a*, *b* and *c* are rational numbers. [CN, ME, R] **20–1 Relations and Functions** S.O. #4. Analyze quadratic functions of the form $y=ax^2+bx+c$ to identify characteristics of the corresponding graph, including: • vertex • domain and range • direction of opening • axis of symmetry • *x*– and *y*–intercepts and to solve problems. [CN, PS, R, T, V] [ICT: C6–4.1, C6–4.3] S.O. 5. Solve problems that involve

S.O. #1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: • representing repeated multiplication, using powers • using patterns to show that a power with an exponent of zero is equal to one • solving problems involving powers. [C, CN, PS, R] 2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents: • $(a^m)(a^n)=a^{m+n}$ • $a^m \div a^n=a^{m-n}, m>n$ • $(a^m)^n=a^{mn}$ • $(ab)^m=a^m b^m$ • $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}, b \neq 0$. [C, CN, PS, R, T] [ICT: P2–3.4] S.O. #5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T] S.O. 5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T] [ICT: P2–3.4] **Grade 8** **Number Strand** S.O. #1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]		quadratic equations. [C, CN, PS, R, T, V] [ICT: C6–4.1] S.O. #6. Solve, algebraically and graphically, problems that involve systems of linear–quadratic and quadratic–quadratic equations in two variables. [CN, PS, R, T, V] [ICT: C6–4.1, C6–4.4] **30–1** **Trigonometry** S.O.# 5. Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians. [CN, PS, R, T, V] [ICT: C6–4.1, C6–4.4] S.O. #6. Prove trigonometric identities, using: • reciprocal identities • quotient identities • Pythagorean identities • sum or difference identities (restricted to sine, cosine and tangent) • double–angle identities (restricted to sine, cosine and tangent). [R, T, V] [ICT: C6–4.1, C6–4.4] **Relations and Functions** S.O. # 10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R] S.O. # 11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree $\leq$ 5 with integral coefficients). [C, CN, ME] S.O. #12. Graph and analyze polynomial functions (limited to polynomial functions of degree $\leq$ 5). [C, CN, T, V] [ICT: C6–4.3, C6–4.4]

Grade 10C **Algebra and Number** 3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]		**20–2 Patterns and Relations** 2. Solve problems that involve quadratic equations. [C, CN, PS, R, T, V] [ICT: C6–4.1, C6–4.3] **30–2 Relations and Functions** 1. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials and binomials). [C, ME, R] 2. Perform operations on rational expressions (limited to numerators and denominators that are monomials and binomials). [CN, ME, R] 3. Solve problems that involve rational equations (limited to numerators and denominators that are monomials and binomials). [C, CN, PS, R] 7. Represent data, using polynomial functions (of degree ≤ 3), to solve problems. [C, CN, PS, T, V] [ICT: C6–4.1, C6–4.3, C6–4.4]

Step 3: Necessary Pre–Requisites

Pre–Requisite Knowledge and Skills
- Collecting "like" terms - Finding perfect squares - Distributive property - Add/ subtract polynomials - multiply / divide monomials - Algebra tiles - Expanding and simplifying
Pre–Requisite Vocabulary
- Like terms - Perfect squares - Distributive law - Expression vs Equation - Polynomial - Monomial, Binomial, Trinomial polynomial - Constant term - Variable - Degree - Coefficient - Simplify - Factors - Multiples

Step 4: Learning Development

Developing Knowledge and Skills
- Binomial multiplication - Factorization (common factoring, difference of squares, trinomial, perfect square trinomials) - Where does this apply to real life situations? - Seeing factors as length/ width of a rectangle? Square? Etc? - Using factors to create area - Using algebra tiles to factor and expand.

Developing Vocabulary	
- All prerequisite vocabulary (from Jr High) - Like terms - Perfect squares - Distributive law - Expression vs Equation - Polynomial - Monomial, Binomial, Trinomial polynomial - Constant term - Variable - Degree	- Coefficient - Simplify - Factors - Multiples - GCF - Prime factorization - LCM - Perfect cube/ perfect root - Radicand/ radical/ index (from previous unit likely) - Decomposition

Predictable Misunderstandings and Errors
Adding/ Subtracting polynomials (collecting like terms) vs multiplying and dividing polynomials (using the powers laws)…. Students will want to change the exponent when adding and subtracting rather than just add/subtract the coefficient.

Step 5: The Big Picture

Essential Understanding:
- Tiles can be used to represent factors and multiples - Factors can be used in a variety of "real life" situations such as: area, volume, construction, design, modeling.
Big Ideas:
- Multiplication and Division are inverses of each other - A rectangle diagram can be used to represent area and factors - Factors and multiples can be used for polynomials - Expressions can be written in various equivalent forms (factored or expanded)
Essential Question:
- How could the understandings from this unit be used in construction and design? Consider the perspectives of potential: Architect? Carpenter? Plumber? Landscape engineers? Etc.

Day 1

1. **Polynomial**

an expression consisting of variables and real numbers . The exponents of the variables must be whole numbers.

Ex: $3x + 11$, $x + \sqrt{2}$, $\sqrt{x}$, $7x^2y - 3$, $\frac{x}{y^2}$ (3rd and 5th are not polynomials).

2. **Coefficient**

the number immediately preceding a variable in a term
(i.e. in $-4x^2 + 7x$, the coefficients are -4 and 7).

3. **Degree of a term**

The sum of the exponents of the variables in the term.

Ex: $9x^4 + 3x^2 + 7$

	$9x^4$	$3x^2$	7
Degree =	4	2	0

4. **Degree of a polynomial.**

Degree of a polynomial is determined by the term with the highest degree.

Ex: $4y^{10} - 6y^7 + 2y^3 - 4$ Degree of this polynomial is 10.

	$4y^{10}$	$6y^7$	$2y^3$	4
Degree =	10	7	3	0

5. **Constant term.**

The term which has no variable.

Ex:
a. $3x^3 + 7x^2 - 6$ (constant term is -6)
b. $7x^5$ (constant term is 0)

6. Classifying polynomials according to the number of terms.

See http://www.youtube.com/watch?v=mRJQtFGj8TY for examples of mono, bi and trinomials.

Number of terms	Name
1	monomial
2	binomial
3	trinomial
4 or more	polynomial

7. Classifying polynomials according to the degree of the polynomial.

Degree	Name	Examples
0	constant	-11
1	linear	$7x + 2$
2	quadratic	$2x^2 - 7x + 3$
3	cubic	$5x^3 + 8x^2 - 9x - 6$

Questions

	Number of terms	Degree	Name of polynomial according to Number of terms	Name of polynomial according to Degree
1. -12				
2. $7x$				
3. $3x + 1$				
4. $x^2 - 5x - 6$				
5. $5x^3 - 3x^2 - x - 10$				
6. $3x^3 - 2x^2 - x$				

Answers

	Number of terms	Degree	Name of polynomial according to Number of terms	Name of polynomial according to Degree
1. -12	1	0	monomial	constant
2. $7x$	1	1	monomial	linear
3. $3x + 1$	2	1	binomial	linear
4. $x^2 - 5x - 6$	3	2	trinomial	quadratic
5. $5x^3 - 3x^2 - x - 10$	4	3	polynomial	cubic
6. $3x^3 - 2x^2 - x$	3	3	trinomial	cubic

II. Addition and subtraction of polynomials.

add/subtract like terms (terms that have the same variable(s) and exponent(s)).

Ex: 1. $(x^2 + 4x - 2) + (2x^2 - 6x + 9) = 3x^2 - 2x + 7$

2. $(4y^2 - 2y + 3) - (3y^2 + 5y - 2) = y^2 - 7y + 5$

Try:

1. $(6a^2 - ab + 4) - (7a^2 + 4ab - 2)$

2. $(2x + y) + (3x - 2y) - (5x - 5y + 2y^2)$

Assignment:

M10C CH3 Poly Intro.docx Contains Blackline Master 3.1c

Day 2 Polynomial Multiplication and Division

Grade 9 Review of Laws or properties of exponents.

1. Product law: $x^a \cdot x^b = x^{a+b}$ i.e. $m^3 \times m^4 = m^7$

   See for further explanations. http://www.youtube.com/watch?v=WBDczW_nvrI

2. Quotient law: $\frac{x^a}{x^b} = x^{a-b}$ i.e. $\frac{m^9}{m^2} = m^7$

   See for further explanations. http://www.youtube.com/watch?v=BcnYAWUT_Y8

3. Power laws: See http://www.youtube.com/watch?v=FyhKX7W5hrE

   a. $(x^m)^n = x^{mn}$ b. $(x^a y^b)^c = x^{ac} y^{bc}$ c. $\left(\frac{x^a}{y^b}\right)^c = \frac{x^{ac}}{y^{bc}}$

   i.e. $(m^3)^4 = m^{12}$ i.e. $(m^3 n^4)^5 = m^{15} n^{20}$ i.e. $\left(\frac{m^3}{n^4}\right)^5 = \frac{m^{15}}{n^{20}}$

Multiplication Monomial by Monomial.

YouTube explanation of mono X mono. http://www.youtube.com/watch?v=1vnCd8pp9Qw

Ex: 1. $(3x^3)(2x^2) = 6x^5$ 2. $(-4a^2b^3)(2ab) = -8a^3b^4$

3. $(8x^3y^2)\left(\frac{1}{2}xy\right) = 4x^4y^3$ 4. $(-4x^3y^2)(2xy^3)(-2xy) = 16x^5y^6$

Division of Monomial by Monomial.

Ex: 1. $\frac{50x}{10x} = 5$ 2. $\frac{20x^2}{5x} = 4x$

3. $\frac{20x^3y^4}{-5x^2y^2}$ $-4xy^2$ 4. $\frac{(3x^2y^2)(5xy)}{-3xy} = -5x^2y^2$

YouTube explanation of mono divided by a mono http://www.youtube.com/watch?v=zB7Z5W_ccZk

Division of Polynomial by Monomial

Rule: Take each term in the numerator (top) and divide it by the term in the denominator (bottom)

Ex: 1. $\frac{4t^2 + 8t}{4t}$ = t + 2, t ≠ 0 $4t^2 \div 4t = t \quad 8t \div 4t = 2 \quad Thus, \; t+2$

2. $\frac{10m^3 + 5m^2 + 15m}{5m}$ = $2m^2 + m + 3$, m ≠ 0

$$10m^3 \div 5m = 2m^2 \quad 5m^2 \div 5m = m \quad 15m \div 5m = 3 \quad Thus, \; 2m^2 + m + 3$$

3. $\frac{6r^4 - 3r^3 + 9r^2}{3r^2}$ = $2r^2 - r + 3$, r ≠ 0

$$6r^4 \div 3r^2 = 2r^2 \quad -3r^3 \div 3r^2 = -r \quad 9r^2 \div 3r^2 = 3$$

4. $\frac{8x^2 + 16x + 24}{8}$ = $x^2 + 2x + 3$

$$8x^2 \div 8 = x^2 \quad 16x \div 8 = 2x \quad 24 \div 8 = 3$$

Distributive Property (Monomial multiplied by a Binomial/Trinomial.

Ex: **1.** $3x^2(2x - 7y + 2)$

$= 6x^3 - 21x^2y + 6x^2$

2. $3(2x^2 - 7) + x(2x + 5) - (8x^2 + 5x - 7)$

$= 6x^2 - 21 + 2x^2 + 5x - 8x^2 - 5x + 7$
$= -14$

3. $9 - 2(3x + 5)$

$= 9 - 6x - 10$
$= -6x - 1$

4. $4a(a + 3) + 2a(a - 1) - a(2a + 4)$

$$4a^2 + 12a + 2a^2 - 2a - 2a^2 - 4a$$
$$4a^2 + 6a$$

5. $3x - 2[3 + 4(2x + 6)] - [2 + 3(x + 1)]$ Hard Question!

$= 3x - 2[3 + 8x + 24] - [2 + 3x + 3]$ Distribute the 4 and the 3.

$= 3x - 6 - 16x - 48 - 2 - 3x - 3$ Distribute the –2 and the –.

$= -16x - 59$ Clean up.

Assignment

M10C CH3 Poly Multiply & Divide.docx

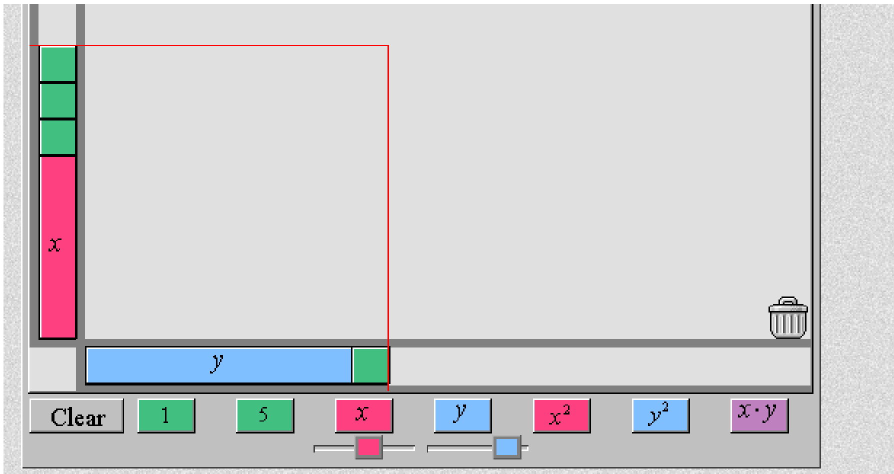
Day 3 Binomial x Binomial (Also known as FOIL)

Multiply $(x+y)(a+b)$ **Two different binomials.**

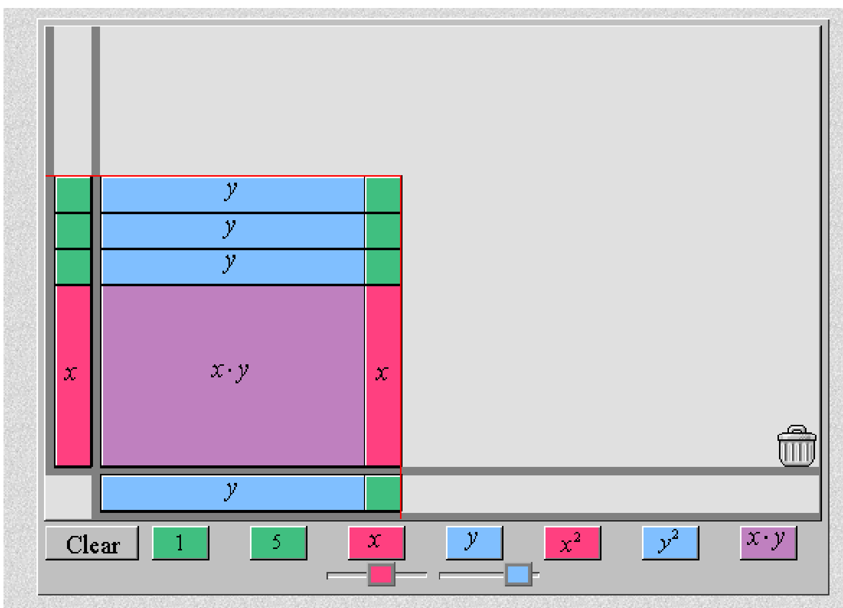
a. $(x+3)(y+1)$

Method 1 – Algebra Tiles

Step 1 – Draw (x + 3) on the vertical axis and (y + 1) on the horizontal axis.



Step 2 – Multiply the vertical and horizontal values and fill in the rectangle.



Step 3 – Determine your final product.

There is 1 xy, 1 x, 3 y's and 3 one's. So, the answer is $xy+x+3y+3$

Method 2 – Area model

Step 1 – Create a square grid with (x + 3) on the vertical axis and (y + 1) on the horizontal axis.

	y	1
x		
3		

Step 2 – Fill in the grid

	y	1
x	xy	x
3	3y	3

Step 3 – Determine your final product.

There is 1 xy, 1 x, 3 y's and a 3. So, the answer is $xy + x + 3y + 3$

Method 3 – FOIL (First, Outer, Inner, Last) The most "mathematical" method.

$(x+3)(y+1)$

$(x+3)(y+1)$ **First** x times y = xy

$(x+3)(y+1)$ **Outer** x times 1 = x

$(x+3)(y+1)$ **Inner** 3 times y = 3y

$(x+3)(y+1)$ **Last** 3 times 1 = 3

You now collect your terms.

There is 1 xy, 1 x, 3 y's and a 3. So, the answer is $xy + x + 3y + 3$

b. $(2x+3)(2y-2)$ **Negatives are grey.**

Algebra Tiles

1	y	y	–1	–1
1	y	y	–1	–1
1	y	y	–1	–1
x	xy	xy	–x	–x
x	xy	xy	–x	–x
	y	y	–1	–1

Determine your final product.

4 xy's, 4 –x's, 6 y's and 6 –1's = $4xy-4x+6y-6$

Area model

Step 1 – Create a square grid with (2x + 3) on the vertical axis and (2y – 2) on the horizontal axis.

	2y	–2
2x		
3		

Step 2 – Fill in the grid

	2y	–2
2x	4xy	–4x
3	6y	–6

Step 3 – Determine your final product.

There are 4 xy's, 4 –x's, 6 y's and 6 –1's = $4xy-4x+6y-6$

FOIL (First, Outer, Inner, Last)

$(2x+3)(2y-2)$

$(2x+3)(2y-2)$ **First** 2x times 2y = 4xy

$(2x+3)(2y-2)$ **Outer** 2x times –2 = –4x

$(2x+3)(2y-2)$ **Inner** 3 times 2y = 6y

$(2x+3)(2y-2)$ **Last** 3 times –2 = –6

You now collect your terms.

There are 4 xy's, 4 –x's, 6 y's and 6 –1's = $4xy-4x+6y-6$

YouTube explanation of **FOIL.** http://www.youtube.com/watch?v=eiDhY3QuqdA

Multiply $(x+y)(x-y)$ **Two binomials that "are the same" but different signs.**

a. $\left(2a-3b\right)\left(2a+3b\right)$ Notice how the 2a and 3b exist in both, but one is + and the other –.

FOIL (First, Outer, Inner, Last)

$\left(2a-3b\right)\left(2a+3b\right)$

$\left(2a-3b\right)\left(2a+3b\right)$ **First** 2a times 2a = $4a^2$

$\left(2a-3b\right)\left(2a+3b\right)$ **Outer** 2a times 3b = 6ab

$\left(2a-3b\right)\left(2a+3b\right)$ **Inner** –3b times 2a = –6ab

$\left(2a-3b\right)\left(2a+3b\right)$ **Last** –3b times 3b = $-9b^2$

You now collect your terms.

There is a $4a^2$, a 6ab, a –6ab and a $9b^2$ = $4a^2-9b^2$

Notice how the Outer and the Inner cancel each other out?
This is always true for $(x+y)(x-y)$
So, a general rule when multiplying binomials of the type $(x+y)(x-y)$ is not FOIL, but
FL. First and Last.

b. $(5x^3 + 4y)(5x^3 - 4y)$

First, let's try **FOIL** (First, Outer, Inner, Last)

$(5x^3 + 4y)(5x^3 - 4y)$

$(5x^3 + 4y)(5x^3 - 4y)$ **First** $5x^3$ times $5x^3$ = $25x^6$

$(5x^3 + 4y)(5x^3 - 4y)$ **Outer** $5x^3$ times –4y = $-20x^3y$

$(5x^3 + 4y)(5x^3 - 4y)$ **Inner** 4y times $5x^3$ = $20x^3y$

$(5x^3 + 4y)(5x^3 - 4y)$ **Last** 4y times –4y = $-16y^2$

Collect our like terms = $25x^6 - 16y^2$

Now, let's try **FL** (First, Last)

$(5x^3 + 4y)(5x^3 - 4y)$

$(5x^3 + 4y)(5x^3 - 4y)$ **First** $5x^3$ times $5x^3$ = $25x^6$

$(5x^3 + 4y)(5x^3 - 4y)$ **Last** 4y times –4y = $-16y^2$

Collect our like terms = $25x^6 - 16y^2$

Thus, when dealing with conjugates (that's what $(x+y)(x-y)$ are called), **you can FOIL or you can FL**.

Multiply $(x+y)^2$ **Squaring a binomial.**

YouTube video and explanation of $(a+b)^2$. http://www.youtube.com/watch?v=ZfEedWRmS5k

a. $(2m+5n)^2$

Let's realize that $(2m+5n)^2$ is no different than $(2m+5n)(2m+5n)$ and we know how to multiple two binomials, **FOIL!**

FOIL (First, Outer, Inner, Last)

$(2m+5n)^2 = (2m+5n)(2m+5n)$

$(2m+5n)(2m+5n)$ **First** 2m times 2m = $4m^2$

$(2m+5n)(2m+5n)$ **Outer** 2m times 5n = 10mn

$(2m+5n)(2m+5n)$ **Inner** 5n times 2m = 10mn

$(2m+5n)(2m+5n)$ **Last** 5n times 5n = $25n^2$

You now collect your terms.

$$4m^2+10mn+10mn+25n^2=4m^2+20mn+25n^2$$

As you can see, squaring a binomial $(x+y)^2$ is just as easy as using FOIL.

But wait! Squaring a binomial $(x+y)^2$ has a shortcut!

Let's try the previous question again.

a. $(2m+5n)^2$

When you are squaring a binomial, here is the shortcut... square the first, square the last, multiply the two and double it. Easy right? Let's see it in practice.

Square the first $2m$ squared = $4m^2$

Square the last 5n squared = $25n^2$

Multiple the two and double it 2m times 5n is 10mn, doubled = 20mn.

Let's collect our like terms = $4m^2+30mn+25n^2$
That's the same answer! Cool huh?

b. $(7c^6-8d^3)^2$

Square the first $(7c^6)^2=49c^{12}$

Square the last $(-8d^3)^2=64d^6$

Multiple the two and double it $7c^6\times-8d^3=-56c^6d^3$ double that gets $-112c^6d^3$

Let's collect our like terms = $49c^{12}-112c^6d^3+64d^6$

YouTube video and explanation of $(a-b)^2$ http://www.youtube.com/watch?v=oWpkZIjBkOU

Assignment

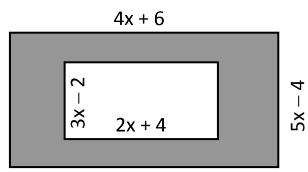
pg 186 #5c,e,f 6a 15ace, 18(tough)

pg 194 #4

Day 4

Applications

Find the area of the shaded region.



$A = A_{Large} - A_{Small}$

$= (4x + 6)(5x - 4) - (2x + 4)(3x - 2)$

$= 14x^2 + 6x - 16$

Multiplication and simplifying of:

1. Mono x Poly
2. Bin x Bin
3. Poly x Poly

Examples

1. $(3x - 2)(x^2 + 5x - 7)$ Put the $(3x - 2)$ into the trinomial

The 3x multiplies into the trinomial and then the –2 multiplies into the trinomial.

$= 3x^3 + 15x^2 - 21x - 2x^2 - 10x + 14$

$= 3x^3 + 13x^2 - 31x + 14$

2. $4a(a + 3) + 2a(a - 1) - a(2a + 4)$

$4a^2 + 12a + 2a^2 - 2a - 2a^2 - 4a$

$4a^2 + 6a$

3. $(2x - 1)(x + 3) - (x - 4)(x - 5)$

$= \left[2x^2 + 5x - 3\right] - \left[x^2 - 9x + 20\right]$

$= 2x^2 + 5x - 3 - x^2 + 9x - 20$

$= x^2 + 14x - 23$

4. (x + 3)(x + 2)(x – 1) Foil the first 2 binomials. Then multiple again.

$= \left(x^2 + 5x + 6\right)\left(x - 1\right)$ The first two binomials are **FOIL**ed

$= x^3 + 5x^2 + 6x - x^2 - 5x - 6$ Distribute the (x–1) into the trinomial

$= x^3 + 4x^2 + x - 6$ Collect like tems.

5. –$(x – 3)^2$ – (x + 5)(x – 5)

$= -\left(x^2 - 6x + 9\right) - \left(x^2 - 25\right)$ Keep the negative out of the squaring of (x–3) and out of FOILing (x+5)(x–5)

$= -x^2 + 6x - 9 - x^2 + 25$ Fire the negatives into the brackets

$= -2x^2 + 6x + 16$ Collect like terms

6. $3t^2 - (3 - 2t)^2 + 5(2t - 1)(2t + 1)$

$= 3t^2 - \left(9 - 12t + 4t^2\right) + 5\left(4t^2 - 1\right)$ square (3–2t) Watch the negative!
FOIL (2t–1)(2t+1)

$= 3t^2 - 9 + 12t - 4t^2 + 20t^2 - 5$ Distribute the negative into the trinomial and the 5 into the binomial.

$= 19t^2 + 12t - 14$ Collect like terms.

7. $2(a + 3)(a - 3) + 3(a + 2)^2$

$= 2\left(a^2 - 9\right) + 3\left(a^2 + 4a + 4\right)$ FOIL (a+3)(a–3) and square (a+2)

$= 2a^2 - 18 + 3a^2 + 12a + 12$ Distribute the 2 and the 3

$= 5a^2 + 12a - 6$ Collect like terms

8. $(3x-4y)^3$

Isn`t this the same as $(3x-4y)(3x-4y)(3x-4y)$ Yes it is! We can do this! It is no different than question 4. FOIL the first two binomials an then distribute the third binomial into the trinomial.

$$
\begin{aligned}
&=(3x-4y)(3x-4y)(3x-4y)\\
&=\left(9x^2-24xy+16y^2\right)\left(3x-4y\right)\\
&=27x^3-72x^2y+48xy^2-36x^2y+96xy^2-64y^3\\
&=27x^3-108x^2y+144xy^2-64y^3
\end{aligned}
$$

Thus, if you ever need to cube a binomial, write it out as the multiplication of 3 binomials and follow proper math :–)

9. $\left(2x+3y-z\right)\left(x-2y+4z\right)$

Here we have a trinomial times a trinomial.
We can look at it in a couple of ways.

<u>Area Model</u>

	x	–2y	4z
2x	$2x^2$	–4xy	8xz
3y	3xy	$-6y^2$	12yz
–z	–xz	2yz	$-4z^2$

Collect like terms = $2x^2-xy+7xz-6y^2+14yz-4z^2$

Distributive Property

Distribute the 2x, 3y and the –z into the trinomial $\left(x-2y+4z\right)$

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ $= 2x^2$

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ = –4xy

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ = 8xz

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ = 3xy

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ $= -6y^2$

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ = 12yz

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ = –xz

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ = 2yz

$\left(2x+3y-z\right)\left(x-2y+4z\right)$ $= -4z^2$ Collect like terms = $2x^2-xy+7xz-6y^2+14yz-4z^2$

Assignment

Page 177 #8
Page 186 #4,7, 8, 11, 13, 14, 19ace, 21ace

Day 5 Common Factors (Prime factorization, GCF, LCM and GCF of a polynomial)

Prime Number - a whole number that has exactly two factors, itself and 1. i.e. 2, 3, 5, 7, 11, etc.

Why aren't 0 or 1 prime numbers? Class discussion.

Prime Factorization - writing a number as a product of its prime factors.

1. Write the prime factorization of the following numbers.

a. 6

2×3

b. 20

2×10 But wait, is 10 a prime number? No! Keep going.

$2\times2\times5 = 2^2\times5$ either is acceptable

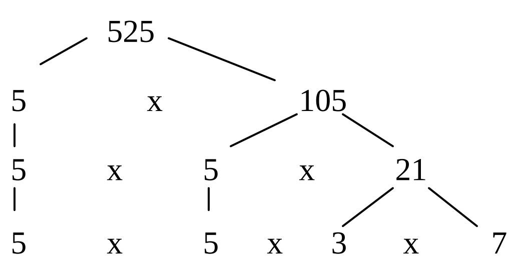
c. 260 What number goes into 260? Many, but I'll use 5

5×52 What goes into 52? 2

$5\times2\times26$

$5\times2\times2\times13 = 2^2\times5\times13$

d. 525 Let's use a factor tree.



$= 3\times5^2\times7$

Question What is common between the words **rake** and **cat**? They both have an **a**.
The **a** would be considered a factor in mathematics.

Question What is the greatest common factor for these numbers?
Note: the GCF (greatest common factor) is the largest number common in the set.

a. 6 and 9

GCF = 3

b. 15, 40, 55

GCF = 5

c. 9, 27, 45

GCF = 9 (yes a GCF can be one of the numbers)

d. $6a \ and \ 9a$ e. $12x \ and \ 18x^3$ f. $8x^2y^3 \ and \ 12xy^4$

GCF = 3a GCF = 6x GCF = $4xy^3$

Thus, when dealing with polynomials, always look to see if there is a GCF for the polynomial.

Question Factor using GCF. Note: Always look for GCF! Always!

1. $5x^2 + 10x$

   $= 5x(x+2)$

2. $x^3 + 7x^2 - 3x$

   $= x(x^2 + 7x - 3)$

3. $8a^2b + 16a^2b^2 + 32a^2bc$

   $= 8a^2b(1 + 2b + 4c)$

4. $x(a + b) + 7(a + b)$

   $= (a+b)(x+7)$

5. $7a(x - 1) + 2(x - 1)$

   $= (x-1)(7a+2)$

6. $3x^2(y + 5) + 2x(y + 5)$

   $= (y+5)(3x^2 + 2x)$

7. $3x(5 - x) - 9(5 - x) + 8y(5 - x)$

   $= 3x(5-x) - 9(5-x) + 8y(5-x)$
   $= (5-x)(3x - 9 + 8y)$

Least Common Multiple or Lowest Common Multiple - the lowest number that a set of numbers are all factors of.

Question Find the LCM for the given set of numbers.

a. 3, 4 and 6 One way to do this is to write out all the multiples of each number

LCM = 12

3, 6, 9, **12**
4, 8, **12**
6, **12**

b. 4, 6, 9

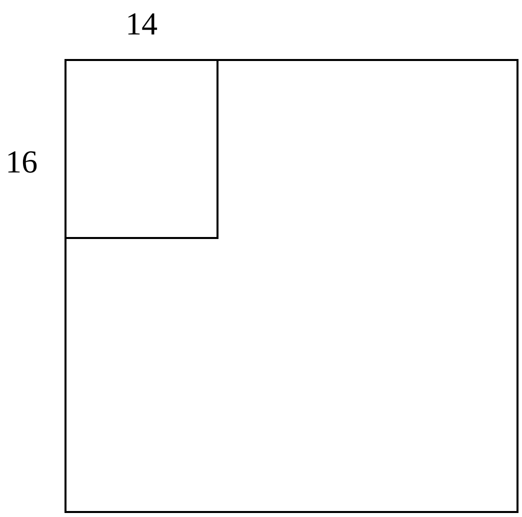
LCM = 36

4, 8, 12, 16, 20, 24, 28, 32, **36**
6, 12, 18, 24, 30, **36**
9, 18, 27, **36**

Real Life Example

Mr. Boychuk has tiles that measure 14 inches by 16 inches that he wishes to make into a square mosaic. What is the smallest square that Mr. Boychuk can create using his 14x16 inch tiles?

Option 1: Draw a diagram



Keep adding 14x16 rectangles until the square is created.

Option 2: List the factors of 14 and 16 to determine the LCM.

14, 28, 42, 56, 70, 84, 98, **112**
16, 32, 48, 64, 80, 96, **112**

Thus, a square 112 inches by 112 inches will fit the 14x16 tiles.

Assignment

Pg 140 #5 ace, 6ace, 7, 10 ace, 11 ac, 21, 22

Pg 155 8-10 (ace in each), 12a, 14ac, 16ace, 18

Day 6 Factoring Difference of Square and Perfect Squares

I. Factoring a difference of squares.

YouTube video on Difference of Squares http://www.youtube.com/watch?v=9hX4dE5sb5o

Let's go back to Day 3 and multiply the following $(3x+2y)(3x-2y)$. If we remember, we could **FOIL**, or we need on **FL**. You need only FL because $(3x+2y)(3x-2y)$ are called conjugate binomials. This means the binomials are identical except one has addition while the other has subtraction.

$(3x+2y)(3x-2y)=9x^2+6xy-6xy-4y^2=9x^2-4y^2$ This was Day 3 so very easy :-)

Let's notice something about the answer and more specifically, what always occurs when you multiply conjugate binomials.

$(x+y)(x-y)=(x^2-y^2)$ There are three things to notice about the answer:

1. The answer is a binomial
2. The answer has subtraction
3. Both terms are perfect squares.

So, if we can multiply binomials, why can't we take the answer and factor it into the original binomials? We can!

Remember, if you notice the three things mentioned above, you have what's called **A Difference of Squares**.

Let's Factor

Ex: 1. $x^2 - 16$ | 2. $x^2 - 25y^2$ | 3. $49x^2 - 64$

$=(x-4)(x+4)$ | $=(x-5y)(x+5y)$ | $=(7x-8)(7x+8)$

Notice how in questions 1-3, they were all binomials, they were all subtraction and each term was a perfect square? Hence, we knew we had **A Difference of Squares**.

4. $x^4 - 81$

$= (x^2 - 9)(x^2 + 9)$ — The $(x^2 - 9)$ is another difference of squares.

$= (x - 3)(x + 3)(x^2 + 9)$

5. $x^2y^3 - y$ (GCF) — Wait! I see a binomial, I see subtraction, but each term is not a perfect square. What do we do? Notice the **Greatest Common Factor**! Always factor the **GCF** out first.

$= y(x^2y^2 - 1)$ — Now the binomial is a perfect square.

$= y(xy - 1)(xy + 1)$

6. $3y^6 - 12d^2$

$= 3(y^6 - 4d^2)$ — Factor out the GCF, 3.

$= 3(y^3 - 2d^2)(y^3 + 2d^2)$ — The binomial was **A Difference of Squares**.

7. $x^{20} - 16$

$= (x^{10} + 4)(x^{10} - 4)$ — Are we done? No! $(x^{10} - 4)$ can be factored.

$= (x^{10} + 4)(x^5 + 2)(x^5 - 2)$

Assignment

Pg 194 #6, 10aceg, 13ac, 21ace

II. Factoring Perfect squares.

Recall from Day 3 that $(a + 3)^2 = a^2 + 6a + 9$
We learned that we could FOIL the binomial, or we could square the first term, square the last term, multiply the two terms and double the product.

Thus, in general $(a+b)^2 = a^2 + 2ab + b^2$

So, when we have a trinomial of the form $a^2 + 2ab + b^2$, it is called a **Perfect Square Trinomial** and can easily be factored back to a single binomial squared.

Ex: 1. $a^2 + 8a + 16 = (a + 4)^2$

2. $9a^2 + 120a + 400 = (3a + 20)^2$

3. $81a^2 - 72ab^3 + 16b^6 = (9a - 4b^3)^2$

Notice in the first three questions how the first and last terms were perfect squares. That's a requirement for **Perfect Square Trinomials.**

4. $28a^2 + 28a + 7$ Wait! 28... 7... these aren't perfect squares. What to do? **GCF**, pull out the 7.

$= 7(4a^2 + 4a + 1)$ 4 and 1, those are perfect squares.

$= 7(2a + 1)^2$

5. $15ax^2 + 90ax + 135a$

$= 15a(x^2 + 6x + 9)$

$= 15(x + 3)^2$

6. $1-12a+36a^2$ This question can be left as is and factored or rearranged in descending order of x, then factored.

Option 1 - Leave as is.

$$1-12a+36a^2$$
$$=(1-6a)^2$$

Option 2 - Rewrite

$$36a^2-12a+1$$
$$=(6a-1)^2$$

These two answers appear different yet they are the same. One answer is negative while the other answer is positive. Think about it, $\sqrt{36}=6 \textit{ and } -6$. Thus, if your answer is squared, then the number being squared could be positive or negative.

Assignment

Pg 194 #7a, 8, 12acd, 13bd

Day 7 **Trinomial Factorization (ax^2+bx+c)**

I. Factoring trinomials ($ax^2 + bx + c, a = 1$)

Recall: $(x + 4)(x + 3) = x^2 + 7x + 12$

– The middle term is the sum of the last terms of each binomial.
– The end term is the product of the last terms of each binomial.

Ex: 1. $x^2 + 8x + 16$

$=(x+4)(x+4)$ Hey, this is a perfect square!

$=(x+4)^2$

2. x^2+5x+6

$=\left(x+?\right)\left(x+?\right)$ We know if must break down to two binomials each with an x. The latter two numbers need to multiply to 6 and add to 5.

Since the 6 and 5 are both positive, we know the ? numbers both need to be positive.

$=\left(x+2\right)\left(x+3\right)$

3. $x^4-8x+15$

$=\left(x^2+?\right)\left(x^2+?\right)$ We know if must break down to two binomials each with an x^2. The latter two numbers need to multiply to 15 and add to –8.

Since the 15 is positive and the –8 is negative, we know the ? numbers both need to be negative.

$=\left(x^2-3\right)\left(x^2-5\right)$

4. $x^{10} + 3x^5 - 28$

$=\left(x^5+?\right)\left(x^5+?\right)$ We know if must break down to two binomials each with an x^5. The later two numbers need to multiply to –28 and add to 3.

Since the –28 is negative we know the ? numbers need to be opposite signs, one positive and one negative.

So what numbers multiply to –28 and add to 3? 7 and –4. **Note:** it can't be –7 and 4.

$=\left(x^2+7\right)\left(x^2-4\right)$

5. $x^2 - 10xy + 21y^2$

$= (x + ?\,y)(x + ?\,y)$ Since the last term has a y^2, there needs to be a y in each binomial.

We now need two numbers that multiply to 21 and add to –10. To multiply out positive but add up negative, both numbers must be negative!

$= (x - 3y)(x - 7y)$

6. $x^{10} - 7x^5y^3 - 30y^6$

$= (x^5 + ?\,y^3)(x^5 + ?\,y^3)$ x^{10} breaks down to x^5 and y^6 breaks down to y^3

What numbers multiply to –30 and add to –7?

$= (x^5 - 10y^3)(x^5 + 3y^3)$

GCF 7. $2x^2 + 10x - 28$ Whoa! What's going on? Think, what is common? The 2 is common! Factor a 2 out.

$= 2(x^2 + 5x - 14)$ Now find two numbers that multiply to –14 and add to 5.

$= 2(x + 7)(x - 2)$

GCF 8. $x^3 - 2x^2y^2 - 8xy^4$ What's common? The x. Factor it out.

$= x(x^2 - 2xy^2 - 8y^4)$ Now find two numbers that multiply to –8 and add to –2.

$= x(x - 4y^2)(x + 2y^2)$

Optional Hints:

1. Both terms positive (middle and last), then both parts are positive.
   $x^2 + 12x + 32 = (x + 8)(x + 4)$

2. Last term positive, middle term negative, then both parts are negative.
   $x^2 - 12x + 32 = (x - 8)(x - 4)$

3. Last term is negative, then one part is negative and the other is positive.
   $x^2 - 4x - 32 = (x - 8)(x + 4)$
   $x^2 + 4x - 32 = (x + 8)(x - 4)$

4. The middle term contains the variables that are found in the first and last terms. The exponents are half of what they are in the first and last terms.

II. Factoring trinomials ($ax^2 + bx + c$, $a \neq 1$)

1. $2x^2 + 13x + 15$ — Here, there is no GCF. So, what are the factors of $2x^2$?

   $=(2x+?)(x+?)$ — Now we need two numbers that multiply to 15.

   The 13 in the middle is not too important as the 2 will influence it.
   Two factors of 15 are 5 and 3. Let's put the 5 and 3 into the ?, but which goes where? We'll try both.

   $=(2x+5)(x+3)$ | $=(2x+3)(x+5)$

   We must FOIL both out to see which is correct.

   $=2x^2+11x+15$ | $=2x^2+13x+15$

   This is NOT correct. | This is correct.

Thus, when the a value in the question ax^2+bx+c is not 1, there is extra work to be done.

2. $5x^2+16x+3$ — Again, no GCF so find the factors of $5x^2$.

   $=(5x+?)(x+?)$ — Now we need the factors of 3. 3 and 1 are good. But again, which goes where? Try both.

   $=(5x+1)(x+3)$ | $=(5x+3)(x+1)$

   We must FOIL both out to see which is correct.

   $=5x^2+16x+3$ | $=5x^2+8x+3$

   This is correct. | This is NOT correct.

3. $12x^4 - 16x^2 + 5$ — This is a tough question. We need the factors of $12x^4$. We know that x^2 will be in both binomials, but the factors of 12 are 2&6, 3&4 and 1&12. Which pair is it? There is no easy answer and sadly we must try them all.
   A good rule of thumb, start in the middle (3&4) and work out.

   $=(6x^2-5)(2x^2-1)$ — This is the correct answer. You can FOIL to verify.

4. $8x^{20} + 10xy - 3y^2$

$= (ax^{10} + by)(cx^{10} - dy)$ — We know that x^{20} breaks down as x^{10} and y^2 breaks down as y.
The factors of 8 are (1&8) and (2&4). The factors of –3 are (3&-1) and (–3&1).
We must now test our options.

$= \left(2x^{10} + 3y\right)\left(4x^{10} - y\right)$ — This is the correct answer. You can FOIL to verify.

5. $10x^2 - 44xy - 30y^2$ — **GCF**. Factor a 2 out.

$= 2(5x^2 - 22x - 15y^2)$

$= 2\left(ax + by\right)\left(cx - dy\right)$ — Find the factors of 5 and –15.

$= 2(5x + 3y)(x - 5y)$ — This is the correct answer. You can FOIL to verify.

Extra Questions if needed:

6. $2x^2 + 11x + 12$
$= \left(2x + 3\right)\left(x + 4\right)$

7. $6m^2 + 13m - 5$
$= \left(3m - 1\right)\left(2m + 5\right)$

8. $4x^2 - 5xy - 6y^2$
$= \left(4x + 3y\right)\left(x - 2y\right)$

9. $6x^2 - 5x - 4$
$= \left(3x - 4\right)\left(2x + 1\right)$

Assignment

Pg 166 #11aceg, 14aceg, 15ac, 17 — $a = 1$

Pg 177 # 13aceg, 15aceg, 17, 19aceg — $a \neq 1$

Day 8

Square Roots and Cube Roots

Square Root - a number, when raise to the power of 2 results in a given number.

$\sqrt{9} = \pm 3$ A Square Root always gives two answers. $3 \times 3 = 9 \quad -3 \times -3 = 9$

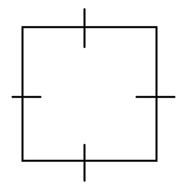
Cube Root - a number, when raised to the power of 3 results in a given number.

$\sqrt[3]{64} = 4$ A cube root only produces one answer. $4 \times 4 \times 4 = 64$

To get a cube root on a TI calculator, hit **MATH** then **4**.

Square - a 2 dimensional object where all sides are equal.

Area = length x width **=** s^2



Cube - a 3 dimension object where all sides are equal. example - a 6 sided die.

Volume = length x width x height = s^3

Examples

1. Find the sides of the square if the area is 289 km ^{2}.

$$A = s^2$$
$$289 = s^2$$
$$\sqrt{289} = \sqrt{s^2}$$
$$s = 17km$$

The negative answer is not applicable so we disregard it.

2. Find the sides of a cube if the area volume is 12167 cm^3.

$$V = s^3$$
$$12167 = s^3$$
$$\sqrt[3]{12167} = \sqrt[3]{s^3}$$
$$s = 23cm$$

Creating Factorable Trinomials.

We know that when we factor ax^2+bx+c, if a = 1 then we need two numbers that multiply to c and add up to b.

$x^2+11x+30=(x+5)(x+6)$ 5 times 6 = 30 and 5+6 = 11

Question: Fill in the box to create factorable trinomials.

a. $x^2+\square x+6$ We need two numbers that multiply to 6. (1&6) or (2&3)

If we use 1&6, then our answer is x^2+7x+6
If we use 2&3, then our answer is x^2+5x+6

But wait! Doesn't $-1\times-6=6$? Or $-2\times-3=6$? Yes! So... we also have....

If we use -1&-6, then our answer is x^2-7x+6
If we use -2&-3, then our answer is x^2-5x+6

Final Answer could be, 7 or 5 or -7 or -5. Any of the four will make the trinomial factorable.

b. $x^2+\square x-14$ We need two numbers that multiply to 6. (1&-14) or (-1&14) or (2,-7) or (-2&7)

If we use 1&-14, then our answer is $x^2-13x-14$
If we use -1&14, then our answer is $x^2+13x-14$
If we use 2&-7, then our answer is $x^2-5x-14$
If we use -2&7, then our answer is $x^2+5x-14$

Final Answer could be, 13 or 5 or -13 or -5. Any of the four will make the trinomial factorable.

c. $x^2 + 4x + \square$ Now we need two numbers that add to 4.

$(1+3)(2+2)(-1+5)(-2+6)$... There are an infinite number!

So choose a pair. Let's try 1 and 3.

$(x+1)(x+3) = x^2 + 4x + 3$

Or we could try –1 and 5.

$(x-1)(x+5) = x^2 + 4x + 4$

So choose a pair of numbers that add to 4 and FOIL away!

d. $x^2 - 5x + \square$ Now we need two numbers that add to –5.
Again, infinite possibilities.

Let's try –8 and 3.

$(x-8)(x+3) = x^2 - 5x - 24$

Some Tricky Factorization

Factor the following:

1. $x^4 - 81$ A difference of squares question.

$= (x^2+9)(x^2-9)$ The (x^2-9) can be factored again!

$= (x^2+9)(x-3)(x+3)$ Rule = factor until you can factor no more.

2. $x^4 - x^2 - 12$ Need two numbers that multiply to –12 and add to –1

$= (x^2-4)(x^2+3)$ First binomial is a difference of squares.

$= (x-2)(x+2)(x^2+3)$

3. $x^4 - 120x^2 - 121$ Need two numbers that multiply to –12 and add to –1

$= (x^2+1)(x^2-121)$ Second binomial is a difference of squares.

$= (x^2+1)(x-11)(x+11)$

Assignment

Page 147 #7, 8	Square and Cube Roots
Page 187 #20a	Volume of Cube
Page 156 #17	Neat Factoring Question
Page 167 #19, 20	Determining b and c values to make trinomials factorable
Page 195 #20	Two step factorization

Day 9 Practicing all types of Factoring.

YouTube video of what to look for when factoring.
http://www.youtube.com/watch?v=KDoHYS4dX1s

I. All types of factoring.

Ex:

1. $x^2 - 7x - 18$

   $= (x-9)(x+2)$

2. $6m^2 + 19mn + 10n^2$

   $= (3m+2n)(2m+5n)$

3. $2x^2 - 8$

   $= 2(x^2-4)$
   $= 2(x-2)(x+2)$

4. $x^3 - 4x^2 + 4x$

   $= x(x^2-4x+4)$
   $= x(x-2)^2$

5. $x(m-2) - 4(m-2)$

   $= (m-2)(x-4)$

6. $3ab - 9ab^2 + 6a^2b$

   $= 3ab(1-3b+2a)$

7. $2y^2 + 7y - 4$

   $= (2y-1)(y+4)$

8. $x^4 - 7x^2y^2 + 12y^4$

   $= \left(x^2-4y^2\right)\left(x^2-3y^2\right)$
   $= \left(x-2y\right)\left(x+2y\right)\left(x^2-3y^2\right)$

Assignment

M10C W CH 3 Factoring.doc

Textbook Review Questions

The textbook has very good questions that can be used for review. You will notice that each set of questions indicated what section of the text it covers. If you need help with a question, you may refer to the section in the text.

Page 198

Questions	Section	
#1-3	3.1	Prime Factors, Lowest Common Multiple & Greatest Common Factor
#6-10	3.2	Square Roots and Cube Roots
#11-14	3.3	Factoring Binomials and Trinomials
#18, 19, 20a, 21	3.5	FOIL and Factoring
#22, 24-26	3.6	FOIL and Factoring
#27-30	3.7	Binomials multiplied by Trinomials and Tri times Tri
#32-35	3.8	Factoring Difference of Squares, Perfect Squares and Applications