Chapter 6: Linear Functions Notes

# Day 1- Slope

We see slope in many different aspects of life. For example: Skate board parks:



Ski Hills:





Theme Parks:



The slope – denoted by m – refers to the steepness of the line. The steeper the line, the greater the slope.

In order to calculate the slope (m) of a line we must find the vertical distance (the rise) and the horizontal distance (the run)

In the following diagram, we are looking to calculate the slope of the line AC. In order to do that, we must first find the vertical change of the line between points A and points C, which we call the rise. On the diagram, the rise is drawn between points B and C. Counting the change from point B up to C, we calculate the rise to be 6.

We must then calculate the horizontal distance of the line, which in this case is the distance from point A to B. This distance is called the run and in our diagram the run is 8.



We can then say that the slope of this line is

$$slope = \frac{rise}{run}$$

$$m = \frac{6}{8}$$
 therefore,  $m = \frac{3}{4}$ 

Typically we leave the slope as a reduced fraction and not in decimal form.

Example:



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When you are given a line, you can select any two points along the line. (Why is it possible to select any two random points along the line?)

Count the following rise and runs on the graph.



### Example:



What do we notice to be different about the graph?

When we go to calculate the slope and we start at the first point to calculate the vertical rise, we instead have to count down to the second point, instead of up. Therefore, when we give the rise, we put is in as a negative.

$$slope = \frac{rise}{run}$$
$$m = \frac{-5}{4}$$

Any line that is decreasing to the right will have a negative slope.

#### Looking at the first example again-



When we talk about the rise (or the change in vertical distance) we are essentially looking at the change in the y coordiates of each of the points. Point A is located at (-3, -2) and point C is located at (5, 4). So the rise is a change from -2 (the y value of point A) to 4 (the y value of point c). Finding the difference in these two values, 4 - (-2) = 6, will give us the Rise.

In the same respect finding the difference in the x values, 5- (-3) = 8, will give you the Run.

Therefore,

$$slope = \frac{rise}{run}$$
$$m = \frac{changeinyvalues}{changeinxvalues}$$
$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

 $y_2$  - the y value of the 2nd point  $y_1$  - the y value of the 1st point  $x_2$  - the x value of the 2nd point  $x_1$  - the x value of the 1st point

In the next example,



What do we notice to be different about the graph?

Let's calculate the slope

The first point is at (-2, 4) therefore,  $x_1 = -2$  and  $y_1 = 4$ . The second point is at (2, -1) therefore,  $x_2 = 2$  and  $y_2 = -1$ .

$$slope = \frac{rise}{run}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{(-1) - 4}{2 - (-2)}$$
$$m = \frac{-5}{4}$$

Does it matter which point you select as point 1 and which you select as point 2?

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Examples:

- 1. Find slopes for each of the following. b. a. c.  $m = \frac{3-3}{5+4} = 0$  $m = \frac{5+1}{4+4} = \frac{6}{8} = \frac{3}{4}$  $m = \frac{-1-4}{2+2} = \frac{-5}{4}$ e. (6.1, 7.8) & (-8.1, 21.2) f. (-2m<sup>2</sup>, 4m) & (19m<sup>2</sup>, 7m) d.  $m = \frac{21.2 - 7.8}{-8.1 - 6.1} = -6.7 \qquad m = \frac{7m - 4m}{19m^2 + 2m^2} = \frac{3m}{21m^2} = \frac{1}{7m}$ m = 0
  - m = undefined or  $\phi$

\*\*\* The slope of a horizontal line (parallel to the x-axis) is 0.

\*\*\* The slope of a vertical line (parallel to the y-axis) is undefined.

\*\*\* Lines rising to the right have a positive slope.

\*\*\* Lines falling to the right have a negative slope.

Determine if the following points are collinear (are on the same line)

Assignment – Pg. 339 # 4. 5, 6ac, 7ac,8, 13 i iv, 14, 15, 16, 17



i.



Meet Slope Man!

When talking about slope, there are only 4 possible slopes, which Slope Man shows us.

Positive, Negative, Zero and Undefined

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Last day we saw two different ways to calculate slope (m).

1. Choosing two points on a graph and counting the rise/run.

Calculate the slope of the following line:

Select two points on the line and count the vertical rise and the horizontal run.

Rise = 1 Run = 1  

$$Slope = \frac{rise}{run}$$
  
 $m = \frac{1}{1}$   
 $m = 1$ 



2. Using two points, finding the  $\frac{\Delta y}{\Delta x}$ 

Using the two following points, calculate the slope. A (2, -4) and B (-2, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{5 - (-4)}{(-2) - 2}$$
$$m = \frac{9}{-4}$$
$$m = \frac{-9}{4}$$

Now, if you were given a slope and a point, could you draw the line?

Example: Given the point C(-3, -3) and a slope of  $\frac{2}{3}$  draw the line on the graph

- 1. Plot point C on the graph
- 2. Count a rise of 2 and a run of 3 and plot that point on the graph
- 3. Plot another point using the slope and then connect the points to create your line



Example: Draw any line with a slope of -4.

- 1. Pick any point on the graph as your initial point
- 2. m = -4 is the same as saying m =  $\frac{-4}{1}$
- Therefore, from your point you are going to count a rise of -4 (down 4 form your point) and a run of one. Plot your 2<sup>nd</sup> point in that spot.



Now, you'll be given the slope and need to find the missing coordinate of one of the points. We will use the slope formula and input all the information we know. Then we simply need to solve for the unknown variable.

Example: Given (4, 2) and (k, 5) with m =  $\frac{3}{5}$ , find k

 $m = \frac{y_2 - y_1}{x_2 - x_1}$  $\frac{3}{5} = \frac{5 - 2}{k - 4}$ 

5(3)=3(k-4)

15=3k-12

27=3k

9=k

Example : Given a line through the point (3, -2) and a slope of  $\frac{2}{5}$ , find another point on the line

i) Graph and choose any point on the line ii) Using the slope formula  $\frac{y+2}{x-3} = \frac{2}{5}$  5y + 10 = 2x - 6  $2x - 5y - 16 = 0 \text{ or } y = \frac{2x - 16}{5}$ 

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We use slope to show how one value changes relative to another value.

Example: When we look at a graph of time vs distance what does the slope represent?

At 1 second the car travelled 20m and at 9 seconds the car travelled 80m.

Therefore, our two points are A (1, 20) and B (9, 80)

$$m = \frac{80 - 20}{9 - 1} = \frac{60}{8} = \frac{15}{2}$$

Rate of change = 7.5 m/s



Example:

The total amount that Thomas earns in a day from his summer job at a gas station is dependent on the number of hours he works. If he works for 8 hours, he earns \$52.

a. Create a table of values for this function

Hours	1	2	3	4	5
Earnings	\$6.50	\$13.00	\$18.50	\$26.00	\$32.50

b. What is the slope of the line? What does the slope represent?

Take 2 points and calculate the slope – (2, 13) and (4, 26)

$$m = \frac{y2 - y1}{x2 - x1}$$
$$m = \frac{26 - 13}{4 - 2}$$

m = 6.5

Slope represents a rate of pay of \$6.50/hour



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c. If Thomas worked for 12 hours on Tuesday, how much would he earn?

d. Thomas agreed to pay his parents \$350/month for rent. What is the minimum number of hours Thomas must work in a month to be able to pay his rent?

\$350 ÷ \$6.50/hr = 54 hrs

<sup>\$6.50/</sup>hr x 12 hours = \$78.00

Example:

For 20 hours of work you were paid \$280. For 32 hours of work you were paid \$412.



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## a. What were you paid per hour for the hours between 20 and 32?

$$m = \frac{412 - 280}{32 - 20} = \frac{132}{12}$$

b. Find your salary for 46 hours. (46, x) 
$$\frac{x - 412}{46 - 32} = 11$$

c. Find your fixed salary. (0, x) 
$$\frac{x - 280}{0 - 20} = 11$$
  
x - 280 = -220  
x = \$60

d. Find the number of hours worked if your salary is \$445	(x, 445)	$\frac{445 - 412}{x - 32} = 11$
		33 = 11x - 352
		385 = 11x
		35 = x

Assignment: pg 340 # 9, 12, 18, 19, 20, 21, 22, 24, 25, 27, 30

### Day 3: Slopes of Parallel and Perpendicular lines AND Investigating Graphs Lab

Calculate the slopes of the following two, parallel lines:

$$\mathcal{L}_{1} = \frac{4}{3}$$
$$\mathcal{L}_{2} = \frac{4}{3}$$



Having been told that the two lines are parallel, what do we discover about slopes of parallel lines? That's right, parallel lines have the same slope.

Calculate the slopes of the following two, perpendicular lines:

$$\mathcal{L}_{1}=\frac{-3}{4}$$

$$\mathcal{L}_{2}=\frac{4}{3}$$



As is the case in this example, and with all perpendicular lines – slopes of perpendicular lines are the negative reciprocals of each other.

## Example:

1. Find slopes of parallel and perpendicular lines to:

	Parallel	Perpendicular $\perp$
a. m = $\frac{3}{5}$	$\frac{3}{5}$	$\frac{-5}{3}$
b. m = 6	6	$\frac{-1}{6}$
c. m = -7/3	$\frac{-7}{3}$	$\frac{3}{7}$
d. m = -8	-8	$\frac{1}{8}$
e. m = 0	0	undefined

### Example:

a. Given A (0, 2), B (-3, -4), C (2, -4) b. Given A (2, 3), B (6, 5), C (-1, 4)

and D (–8, 1), show that  $\overline{
m AB} ot \overline{
m CD}$  .

and D (3, 6), is  $\overline{AB} \parallel \overline{CD}$ ?

$$m_{AB} = \frac{-4-2}{-3-0} = \frac{-6}{-3} = 2$$
Negative reciprocals
$$m_{AB} = \frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$$

$$m_{CD} = \frac{1+4}{-8-2} = \frac{5}{-10} = \frac{-1}{2}$$

$$m_{CD} = \frac{6-4}{3+1} = \frac{2}{4} = \frac{1}{2}$$
Yes

Example:

A line AB passes through the points A (2, 8) and B (-3, -5).

- A. Sketch the line
- B. Sketch a line parallel to line AB
- C. Sketch a line perpendicular to line AB



Example:

Determine if line CD and line EF are parallel, perpendicular or neither. Line CD passes through points C (0, 5) and D (5, -8) and line EF passes through points E (-4, -6) and F (3, 3).



The two lines are neither parallel or perpendicular

Example:







Trapezoid

Parallelograms

Rectangle

Square

Quadrilateral: 4 sides  $\rightarrow$  # 1, 2, 3, 4, 5, 6

Trapezoid: at least one set of // sides  $\rightarrow$  # 2, 3, 4, 5, 6

Parallelogram: two sets of parallel sides  $\rightarrow$  # 3, 4, 5, 6

Rhombus: two sets of parallel sides and all sides equal length  $\rightarrow$  4, 6

Rectangle: two sets of parallel sides,  $90^{\circ}$  corners  $\rightarrow$  5, 6

Square: two sets of parallel sides, all sides equal,  $90^{\circ}$  corners  $\rightarrow 6$ 

Examples:

1. Determine type of quadrilateral given A(2, 5), B(1, 0), C(-4, -1), D(-3, 4).



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#### Ans: parallelogram specifically a rhombus

2. A = (-3, 4), B = (-1, 7), C = (5, 3), D = (3, 0).



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#### Ans: rectangle

Assignment: Pg. 349 #3ad, 4bc, 5cd, 6bc, 7, 8bd, 9cd, 10a, 11, 13, 16, 19

## Do 6.3 – Investigating Graphs of Linear Functions- Do page 356 together

### Day 4 – Quiz and Slope-Intercept Form

We are now going to look at describing linear functions using equations. Last day we looked at entering equations, such as y=2x+5 into our calculators and saw that they produced a straight line graph. We noticed in our investigation, that changing the value of the 2 in the equation affected the steepness, or slope of the graph and that adjusting the 5 moved the graph vertically up or down on the Cartesian plane.

Writing the equation in this way is called the Slope-Intercept Form. The slope intercept form is:

y = mx+b where m=slope and b= y intercept

That is to say that in the above example, y = 2x+5, the graph has a slope of 2 and crosses the y axis at 5. Using that information, we could then graph the equation.

1)Place the first point at the y intercept of 5.

2) From that point count out your slope of 2 (rise of 2 and run of 1) and plot your second point.



http://www.zonalandeducation.com/mmts/functionInstitute/linearFunctions/lsif.html

Example: Given the following equations, find the slope and the y intercept and sketch the graphs.

a) y = -5x + 4	<b>b)</b> $y = \frac{3}{4}x + 1$	c) y = -4x	
m = -5	$m=\frac{3}{4}$	m = -4	
b = (0, 4)	b= (0, 1)	b = (0, 0)	

d) y = -3 e) y = x - 1.25 f) x = 12

m = 0

m = 1

m = undefined

b = (0, -1.25)



b = (0, -3) No y intercept

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Example: Given the following graphs, find the equation of the lines in slope intercept form.



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- $\mathcal{L}_1$ ) y = x + 1
- $\mathcal{L}_2$ ) y = -3/4x 2
- *L*<sub>3</sub>) x = -8
- $\mathcal{L}_4$ ) y = 8
- $\mathcal{L}_5$ ) y = 3x
- $\mathcal{L}6$ ) y = -5x + 7

Example: Complete the following chart

Table of Values	Graph	Slope (m)	y-intercept (b)	Equation y = mx + b
x         y           0         1           1         3           2         5	Created with a trial version n			
х у 	Created with a trial version o			
x         y           0         -1           2         0           4         1	Created with a trial version of			
x y	Created with a trial version of			γ = -2x - 1

Example: The SCIL group at Salisbury rent a portable dunk tank as a fundraiser activity. Students pay for the chance to hit a target and dunk Mr. Chandler into the water. The dunk tank costs \$300 to rent and the SCIL group intends to charge \$1.50 per ball at the event.

a) Write an equation for the profit, P dollars, on the sale of balls (B) at the event.

We find profit by taking the Income and subtracting the Expenses

P = Income – Expense

The income earned is going to be \$1.50/ball x number of balls purchased

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Income = 1.50(B)
```

Therefore,

b) Suppose 50 balls were purchased to try and dunk Mr. Chandler. How much profit would the group raise?

Substitute the number of balls in for 'B'

```
P = 1.5B - 300
P = 1.5(50) - 300
P = $-225.00
```

The group would lose \$225.00 if only 50 balls were purchased

c) If the group raised \$350 profit, how many balls were purchased?

Substitute \$350 in for P and solve for B

$$P = 1.5B - 300$$

$$350 = 1.5B - 300$$

$$350 + 300 = 1.5B$$

$$650 = 1.5B$$

$$\frac{650}{1.5} = B$$

$$633 = B$$

The group would need to sell 633 throws in order to gain a profit of \$350.00

d) How many balls need to be purchased in order for the group to break even?

In order to break even the Profit = 0. Therefore, substitute 0 in for P and solve

P = 1.5B - 3000 = 1.5B - 3000 + 300 = 1.5B300 = 1.5B $\frac{300}{1.5} = B$ 200 = B

The group would need to sell 200 balls in order to break even

Example: Jane has selected a hotel for her wedding reception. The cost involves a fee for the deluxe ballroom and a buffet charge that depends on the number of guests. This is shown in the following table.

Number of Guests	Cost (\$)
0	425
25	1800
50	3175
100	5925

a) Sketch a graph of the data in the table



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b) What are the slope and y intercept? What do the slope and y intercept represent?

Using two points - (0, 425) and (25, 1800)

$$slope = \frac{\Delta y}{\Delta x}$$
$$m = \frac{1800 - 425}{25 - 0}$$
$$m = \frac{1375}{25}$$
$$m = 55$$

y-intercept – (0, 425)

c) Write an equation that describes the relationship between the cost and the number of guests in slope-intercept form.

y = mx + b

- C = 55x + 425, where x is the number of guests
- d) What is the cost for 140 guests?
- C = 55x + 425
- C = 55(140) + 425
- C = \$8125.00

e) Jane would like to spend no more than \$15 000. What is the maximum number of guests that can attend?

C = 55x + 425 15 000 = 55x + 425 15 000 - 425 = 55x 14 575 = 55x x = 265

Assignment- Pg. 362 # 4bdef, 5ace, 6ab, 7def, 8, 9, 11, 12cd, 13, 14, 15, 16, 17a, 22, 23

### Day 5: Slope-Point Form of the Equation

We saw that slope is calculated using the formula

$$slope = \frac{rise}{run}$$
$$m = \frac{changeinyvalues}{changeinxvalues}$$
$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If we then multiplied both sides of the equation by  $(x_2 - x_1)$ , thus not changing the equation only rearranging it, we would have

$$(x_2 - x_1)m = (x_2 - x_1)(\frac{y_2 - y_1}{x_2 - x_1})$$
$$(x_2 - x_1)m = y_2 - y_1$$

This equation is called the slope-point form of a non-vertical line through point  $(x_1, y_1)$ 

The slope-point form is commonly written as  $y - y_1 = m(x - x_1)$ 

http://www.zonalandeducation.com/mmts/functionInstitute/linearFunctions/lpsf.html

Example: a) Write the equation of a line through (-2, 5) with a slope of -3.

Substitute -3 for m and the coordinates of the point (-2, 5) for  $(x_1, y_1)$ 

 $y - y_1 = m(x - x_1)$ 

$$y - 5 = -3(x - (-2))$$

y - 5 = -3(x + 2)

The equation in slope-point form is y - 5 = -3(x + 2)

b) Express the equation in slope-intercept form

To express in slope intercept form we need to isolate y

y - 5 = -3(x + 2) y = -3(x + 2) + 5 y = -3x - 6 + 5 y = -3x - 1The slope-intercept form is y = -3x - 1 Example: Use slope-point form to write an equation of the line through (3, -4) and (5, -1)

Since we are given two points we can calculate the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-1 + 4}{5 - 3}$$
$$m = \frac{3}{2}$$

Now we have a slope and can use either point to input into our equation

y - y<sub>1</sub> = m(x- x<sub>1</sub>) or y - y<sub>1</sub> = m(x- x<sub>1</sub>)  
y + 4 = 
$$\frac{3}{2}$$
 (x - 3) y + 1 =  $\frac{3}{2}$  (x - 5)

## How can you verify that these equations are equivalent?

Assignment: Pg. 372 # 4abf, 5cd, 7bc, 8, 9i ii, 10, 11ab, 13, 15, 18, 19, 22, 24

## Day 6: General Form

The general form of a linear equation is Ax + By + C = 0, where A is a whole number and B and C are integers.

http://www.zonalandeducation.com/mmts/functionInstitute/linearFunctions/lgf.html

You can convert a linear equation from one form to another by applying the rules of algebra.

Example: Convert y =  $\frac{-2}{3}$  x + 6 into general form

$$y = \frac{-2}{3}x + 6$$
  

$$3(y) = 3(\frac{-2}{3}x + 6)$$
  

$$3y = -2x + 18$$
  

$$2x + 3y = 18$$
  

$$2x + 3y - 18 = 0$$

Example: Convert x - 4y + 2 = 0 into slope-intercept form

x - 4y + 2 = 0

x+2=4y $\frac{x}{4}+\frac{2}{4}=\frac{4y}{4}$  $\frac{1}{4}x+\frac{1}{2}=y$ 

Example: Convert  $y + 4 = \frac{3}{2}(x - 3)$  into general form

$$y+4 = \frac{3}{2}(x-3)$$
  
2(y+4) = 2 $\left(\frac{3}{2}(x-3)\right)$   
2y+8 = 3(x-3)  
2y+8 = 3x-9  
0 = 3x-2y-17

#### Finding x and y intercepts

We know that at the x intercept the y value is zero. Therefore if looking for the x-intercept we can substitute O into our equation in the place of y and we are left with only the variable x in our equation. If we solve for x, we find the value x when y is zero. In other words we find the x-intercept.

The same works for the y-intercept. At the y-intercept, the x value is zero. By substituting zero into the equation for x, we can solve for y to find the y value

Example: For the linear equation 2x - 3y - 6 = 0

a) State the x-intercept of the graph

$$2x - 3y - 6 = 0$$
  
 $2x - 3(0) - 6 = 0$   
 $2x - 6 = 0$   
 $2x = 6$   
 $x = 3$ 

The x intercept is (3, 0)

b) State the y-intercept of the graph

2x - 3y - 6 = 0 2(0) - 3y - 6 = 0 -3y - 6 = 0 -3y = 6y = -2

The y intercept is (0, -2)

c) Use the intercepts to graph the line



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Example: Given the linear equation 3x + 4y + 12 = 0, find the slope and the x and y intercepts.

$$3x + 4y + 12 = 0$$
  

$$4y = -3x - 12$$
  

$$y = \frac{-3}{4}x - 3$$
  

$$3x + 4(0) + 12 = 0$$
  

$$3x + 12 = 0$$
  

$$3x = -12$$
  

$$x = -4$$

m = 
$$\frac{-3}{4}$$
 b= (0, -3) x-int = (-4, 0)

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Example: When an equation is written in general form Ax + By + C = 0, what is the effect on the graph if

a) A = 0

x + 4.5 = 0

x = -4.5



Assignment- Pg. 384 # 4, 5ab, 6cd, 7a, 8i ii, 10, 12ac, 13cd, 16, 18ac, 21, 23, 26, 27