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**Math 30-2 Notes**

**Chapters 4 - 5**

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**Math 30-2 Notes**

**Chapter 4: Rational Expressions and Equations**

**(pp. 213 – 269)**

**4.1 Equivalent Rational Expressions** (pp. 216 – 224)

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| **Term** | **Definition / Equation** | **Example** |
| Rational Number | **A ratio of two integers in which the denominator does not equal zero.** | $\frac{5}{7}$ **is a rational number.** |
| Rational Expression | **A fraction that has a polynomial for both the numerator and the denominator.** **(The denominator must not equal zero.)**  | The following are examples of rational expressions:$$\frac{2x}{3x+4}, \frac{4x^{2}+3x+1}{9}, \frac{1}{5x^{1000}-219x^{3}+2}$$The following are NOT examples of rational expressions:$$\frac{\sqrt{x+2}}{3}, \frac{5^{x}}{11x}, \frac{\left|3x+2\right|}{2x-1}$$ |
| Non-Permissible Value (NPV) | The value of a variable that makes the **denominator of a rational expression equal to zero.**If the denominator contains a binomial, the non-permissible values can be determined from the **factored form of the binomial.**All non-permissible values must be stated as restrictions on the variable to ensure that the expression is defined. | 1) $\frac{10}{15+x}$ is undefined when $x=-15$**, because this creates a zero denominator. Thus,** $x=-15$ **is a non-permissible value for** $\frac{10}{15+x}$**.**2) Determine the non-permissible values for $\frac{4x-1}{3x^{2}-9x}$.**The denominator is** $3x^{2}-9x$**. Factor the denominator, setting it equal to zero, and examine each factor.**$$3x^{2}-9x=0$$$$3x\left(x-3\right)=0$$**3**$x=0, x-3=0$$$x=0, x=3$$**The non-permissible values are** ***x* = 0 and *x* = 3.** |
| Equivalent Rational Expression | A new rational expression that has the same meaning as the original expression, obtained by multiplying or dividing both the numerator and denominator by the same factor. Multiplying numerator and denominator by the same factor or dividing numerator and denominator by a common factor is like multiplying the entire expression by **1.** Substitution can show that two rational expressions are not equivalent, but cannot prove equivalence. For rational expressions to be equivalent, they must be equal **for ALL permissible values of the variable.**Note: Make sure that you do not introduce factors that require **a new restriction. Non-permissible values must be stated, and should be consistent – the same for the original expression and the final expression.** | 1) A rational expression equivalent to $\frac{3}{7}$ is:$$\frac{3}{7}∙\frac{3}{3}=\frac{9}{21}$$2) A rational expression equivalent to  $\frac{4x-2}{7x}, x\ne 0$ is:$$\frac{4x-2}{7x}∙\frac{x^{2}}{x^{2}}=\frac{4x^{3}-2x^{2}}{7x^{3}}, x\ne 0$$3) A rational expression equivalent to  $\frac{4x-2}{2x-1}, x\ne \frac{1}{2}$ is:$$\frac{4x-2}{2x-1}=\frac{2(2x-1)}{2x-1}=2, x\ne \frac{1}{2}$$4) A rational expression equivalent to $\frac{2-2x}{4}$ is:$$\frac{2-2x}{4}=\frac{2(1-x)}{2∙2}=\frac{1-x}{2}$$5) Determine whether $\frac{9}{3x-1}$ and $\frac{-18}{2-6x}$ are  equivalent expressions.To be equivalent, the expressions must have the **same restrictions, and must be identical when simplified.**Determine the restrictions, factoring the denominator if needed.Simplify each expression, maintaining the restrictions.

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| $$\frac{9}{3x-1}, x\ne \frac{1}{3}$$ | $$\frac{-18}{2-6x}=\frac{-18}{2\left(1-3x\right)},$$$$ x\ne \frac{1}{3}$$$$\frac{-2∙9}{2\left(1-3x\right)}=\frac{-9}{1-3x}$$$$=\frac{9}{3x-1}, x\ne \frac{1}{3}$$ |

**The two rational expressions are equivalent for all permissible values.** |

**4.2 Simplifying Rational Expressions** (pp. 225 – 231)

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| **Term** | **Definition / Equation** | **Example** |
| Simplified Rational Expression | A rational expression in which the numerator and the denominator have **no common factors.** In other words, the greatest common factor of the numerator and the denominator must be **1 for a rational expression in simplified form.**Simplify a rational expression by **factoring the numerator and the denominator, and then dividing both by their greatest common factor (GCF).**Always state the non-permissible value(s) of the variables as restrictions **before simplifying a rational expression to avoid losing information about the non-permissible value(s) of the variable.** | 1) Simplify $\frac{18x^{4}}{36x^{7}}$.**State restrictions before simplifying.**$$\frac{18x^{4}}{36x^{7}}, x\ne 0$$**Factor numerator and denominator to identify common factors.**$$\frac{18x^{4}}{36x^{7}}=\frac{18x^{4}}{2x^{3}(18x^{4})}=\frac{1}{2x^{3}}, x\ne 0$$2) Simplify $\frac{3a^{3}-3a^{2}}{-12a+12}$.**State restrictions before simplifying.****Denominator:** $-12a+12\ne 0$$$-12(a-1)\ne 0\rightarrow a\ne 1$$**Factor numerator and denominator to identify common factors.**$$\frac{3a^{2}(a-1)}{-12(a-1)}=\frac{a^{2}(3)}{-4(3)}=\frac{a^{2}}{-4}=\frac{-a^{2}}{4}, a\ne 1$$ |
| Inadmissible Value for a Variable in a Rational Expression | A value for a variable that **does not work in the context of the problem.** | If $\frac{100}{x}$ is a rational expression that represents the length of a rectangle with width *x*, then:***x* = 0 is a non-permissible value and *x* < 0 represents the inadmissible values. A rectangle with negative width does not make sense.** |

**4.3 Multiplying and Dividing Rational Expressions** (pp. 232 – 239)

* The strategies used to multiply and divide rational numbers can be used to multiply and divide rational expressions. That is, if *A*, *B*, *C*, and *D* are polynomials, then:

$$\frac{A}{B}∙\frac{C}{C}=\frac{AC}{BD}, B, D\ne 0$$

$$\frac{A}{B}÷\frac{C}{D}=\frac{A}{B}∙\frac{D}{C}=\frac{AD}{BC}, B, D, and C\ne 0$$

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| **Example 1. Simplifying a product.**Simplify the following product:$$\frac{12x^{3}}{3x^{2}+6x}∙\frac{4x^{3}+8x^{2}}{5}$$ |
| **Strategy / Step** | **Solution** |
| Factor the numerators and denominators of both expressions, if possible. | $$\frac{12x^{3}}{3x^{2}+6x}∙\frac{4x^{3}+8x^{2}}{5}=\frac{12x^{3}}{3x(x+2)}∙\frac{4x^{2}(x+2)}{5}$$ |
| Identify the non-permissible values of the variable. To do so, determine all the values that make the denominators equal to zero in the factored product. | **Non-permissible values occur when the denominators equals zero.**$$3x\left(x+2\right)\left(5\right)=0$$$$3x=0, x+2=0$$$$x=0, x=-2$$**The restrictions on the variable are:**$$x\ne -2, 0$$ |
| Multiply the numerators and multiply the denominators, writing each product as a single rational expression. | $$\frac{12x^{3}∙4x^{2}(x+2)}{3x\left(x+2\right)(5)}=\frac{48x^{5}(x+2)}{15x(x+2)}$$ |
| Simplify using common factors. | $$=\frac{16x^{4}\left(3x\right)(x+2)}{5\left(3x\right)(x+2)}=\frac{16x^{4}}{5}$$ |
| Write the product, stating the restrictions on the variable. | $$\frac{16x^{4}}{5}, x\ne -2, 0$$ |

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| **Example 2. Simplifying a quotient.**Simplify the following quotient:$$\frac{x^{3}+x^{2}}{16}÷\frac{x^{2}+x}{20x-10}$$ |
| **Strategy / Step** | **Solution** |
| Factor the numerators and denominators of both expressions, if possible. | $$\frac{x^{3}+x^{2}}{16}÷\frac{x^{2}+x}{20x-10}=\frac{x^{2}(x+1)}{16}÷\frac{x(x+1)}{10(2x-1)}$$ |
| Identify the non-permissible values of the variable. To do so, determine all the values that make the denominators equal to zero in the factored quotient. For division, remember to consider both the numerator and the denominator of the divisor. | **Non-permissible values occur when either of the denominators or the numerator of the divisor equal zero.**$10\left(2x-1\right)=0$ **or** $x\left(x+1\right)=0$$2x+1=0$ **or** $x=0$ **or** $x+1=0$$$x=-\frac{1}{2}, 0, -1$$**The restrictions on the variable are:**$$x\ne -1,-\frac{1}{2}, 0$$ |
| Multiply by the reciprocal of the divisor. | $$\frac{x^{2}(x+1)}{16}∙\frac{10(2x-1)}{x(x+1)}$$ |
| Multiply the numerators and multiply the denominators, writing each product as a single rational expression. | $$=\frac{10x^{2}\left(x+1\right)(2x-1)}{16x(x+1)}$$ |
| Simplify using common factors. | $$=\frac{5x\left(2x\right)\left(x+1\right)(2x-1)}{8\left(2x\right)(x+1)}=\frac{5x(2x-1)}{8}$$ |
| Write the quotient, stating the restrictions on the variable. | $$\frac{5x(2x-1)}{8}, x\ne -1,-\frac{1}{2},0$$ |

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| **Example 3. Simplifying a rational expression containing several binomials.**Simplify the following expression:$$\frac{4x^{2}-1}{x+2}÷\frac{4x^{2}+2x}{8x^{2}-32}$$ |
| **Strategy / Step** | **Solution** |
| Factor the numerators and denominators of both expressions, if possible. | $$\frac{\left(2x-1\right)(2x+1)}{(x+2)}÷\frac{2x(2x+1)}{8(x^{2}-4)}=\frac{\left(2x-1\right)(2x+1)}{(x+2)}÷\frac{2x(2x+1)}{8\left(x-2\right)(x+2)}$$ |
| Identify the non-permissible values of the variable. To do so, determine all the values that make the denominators equal to zero in the factored quotient or product. For division, remember to consider both the numerator and the denominator of the divisor. | **Non-permissible values occur when either of the denominators or the numerator of the divisor equal zero.**$x+2=0$ **or** $8\left(x-2\right)\left(x+2\right)=0$ **or** $2x\left(2x+1\right)=0$$x+2=0$ **or** $x-2=0$ **or** $x+2=0$ **or** $2x=0$ **or** $2x+1=0$$$x=-2, 2, 0, -\frac{1}{2} $$**The restrictions on the variable are:**$$x\ne -2,-\frac{1}{2},0, 2$$ |
| Multiply by the reciprocal of any divisors. | $$\frac{\left(2x-1\right)(2x+1)}{(x+2)}∙\frac{8\left(x-2\right)(x+2)}{2x(2x+1)}$$ |
| Multiply the numerators and multiply the denominators, writing each product as a single rational expression. | $$=\frac{8\left(2x-1\right)\left(2x+1\right)\left(x-2\right)(x+2)}{2x\left(x+2\right)(2x+1)}$$ |
| Simplify using common factors. | $$=\frac{4(2)\left(2x-1\right)\left(2x+1\right)\left(x-2\right)(x+2)}{x(2)\left(x+2\right)(2x+1)}=\frac{4\left(2x-1\right)(x-2)}{x}$$ |
| Write the quotient, stating the restrictions on the variable. | $$\frac{4\left(2x-1\right)(x-2)}{x}, x\ne -2,-\frac{1}{2},0, 2$$ |

**4 Mid-Chapter Review**  (pp. 240 – 241)

Q1) Why do you need to state the non-permissible values for a rational expression?

A1) **Division by zero is undefined in the real number system. If there are variables in the denominator of a rational expression, it is necessary to ensure that they do not take on any values that would make the denominator zero. If a variable in the denominator ever takes a value that makes the denominator equal to zero, then the rational expression contains division by zero, making the expression undefined. To avoid this problem, state the non-permissible values at the beginning of your solution as restrictions on the variable.**

Q2) Why can you NOT simplify a rational expression by dividing like terms as shown below?

$$\frac{4x+5}{4x+2}=\frac{5}{2}$$

A2) **Rational expressions are like fractions. To write a fraction in simplest terms, you divide the numerator and denominator by common factors. You can only simplify a rational expression by dividing common factors, not by dividing common terms.**

Q3) When is it necessary to analyze the numerator of a rational expression to determine non-permissible values?

A3) **The only time you would analyze a numerator to determine non-permissible values is when you are dividing two rational expressions. Non-permissible values occur when there is the possibility of dividing by zero. When there is a quotient of two rational expressions, the numerator of a divisor may give rise to a non-permissible value or values.**

**4.4 Adding and Subtracting Rational Expressions**  (pp. 244 – 252)

* The strategies for adding and subtracting rational expressions are the same as the strategies for adding and subtracting rational numbers.
* When rational expressions are added or subtracted, they must have a common denominator.

$$\frac{A}{B}+\frac{C}{D}=\frac{AD+CB}{BD}, B, D\ne 0$$

$$\frac{A}{B}-\frac{C}{D}=\frac{AD-CB}{BD}, B, C\ne 0$$

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| **Example 1. Adding rational expressions.**Simplify the sum:$$\frac{3}{6x^{2}}+\frac{1}{4x}$$ |
| **Strategy / Step** | **Solution** |
| Factor the numerators and denominators of both expressions, if possible. State any restrictions on the variable before simplifying either expression. | $$\frac{1(3)}{2x^{2}(3)}+\frac{1}{4x}, x\ne 0$$$$\frac{1}{2x^{2}}+\frac{1}{4x}, x\ne 0$$ |
| Determine the lowest common denominator (LCD). The LCD is the product of all the common factors and all the unique factors of the denominators. The LCD is not always the product of all the denominators. | **Since** $4x^{2}$ **is divisible by** $4x$ **and** $2x^{2}$**, it is the LCD.** If we directly follow the steps to the left to determine the LCD:* Factors of first denominator: **2 and *x* and *x***
* Factors of second denominator: **2 and 2 and *x***
* Common factors: **2 and *x***
* Unique factors: **2 and *x* (since only one 2 is a factor of the first denominator and only one x is a factor of the second denominator)**

Then the LCD is $2∙2∙x∙x=4x^{2}$ **(product of all common factors and unique factors)** |
| Rewrite each rational expression as an equivalent expression with the LCD as the denominator. | $$\frac{1}{2x^{2}}∙\frac{2}{2}+\frac{1}{4x}∙\frac{x}{x}=\frac{2}{4x^{2}}+\frac{x}{4x^{2}}$$ |
| Add the numerators of the equivalent expressions. | $$=\frac{2+x}{4x^{2}}$$ |
| Simplify the rational expression and restate the restrictions on the variable. The non-permissible values of the variable in the simplified expression are the combination of the non-permissible values of the original expression. | $$\frac{2+x}{4x^{2}}, x\ne 0$$**To ensure that the final expression is fully simplified, double-check that there are no common factors in the numerator and denominator. This particular expression is fully simplified.** |

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| **Example 2. Subtracting rational expressions.**Simplify the difference:$$\frac{6}{n-3}-\frac{4}{n+2}$$ |
| **Strategy / Step** | **Solution** |
| Factor the numerators and denominators of both expressions, if possible. State any restrictions on the variable before simplifying either expression. | $$\frac{6}{n-3}-\frac{4}{n+2}, n\ne 3, -2$$ |
| Determine the lowest common denominator (LCD). The LCD is the product of all the common factors and all the unique factors of the denominators. The LCD is not always the product of all the denominators. | If we directly follow the steps to the left to determine the LCD:* Factor(s) of first denominator: ***n – 3***
* Factor(s) of second denominator: ***n + 2***
* Common factors: **none**
* Unique factors: ***(n – 3)* and *(n + 2)***

Then the LCD is the product of the common and unique factors: $\left(n-3\right)(n+2)$ |
| Rewrite each rational expression as an equivalent expression with the LCD as the denominator. | $$\frac{6}{n-3}-\frac{4}{n+2}=\frac{6}{(n-3)}∙\frac{(n+2)}{(n+2)}-\frac{4}{(n+2)}∙\frac{(n-3)}{(n-3)}$$$$=\frac{6(n+2)}{\left(n-3\right)(n+2)}-\frac{4(n-3)}{\left(n-3\right)(n+2)}$$ |
| Determine the difference of the numerators of the equivalent expressions. | $$=\frac{6\left(n+2\right)-4(n-3)}{\left(n-3\right)(n+2)}=\frac{6n+12-4n+12}{\left(n-3\right)(n+2)}=\frac{2n+24}{\left(n-3\right)(n+2)}=\frac{2(n+12)}{\left(n-3\right)(n+2)}$$ |
| Simplify the rational expression and restate the restrictions on the variable. The non-permissible values of the variable in the simplified expression are the combination of the non-permissible values of the original expression. | $$\frac{2(n+12)}{\left(n-3\right)(n+2)}, n\ne -2, 3$$**To ensure that the final expression is fully simplified, double-check that there are no common factors in the numerator and denominator. This particular expression is fully simplified.** |

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| **Example 3. Using a factoring strategy to simplify a rational expression.**Simplify the difference:$$\frac{2x}{x^{2}-1}-\frac{4}{x-1}$$ |
| **Strategy / Step** | **Solution** |
| Factor the numerators and denominators of both expressions, if possible. State any restrictions on the variable before simplifying either expression. | $$\frac{2x}{\left(x-1\right)(x+1)}-\frac{4}{x-1}, x\ne -1, 1$$ |
| Determine the lowest common denominator (LCD). The LCD is the product of all the common factors and all the unique factors of the denominators. The LCD is not always the product of all the denominators. | If we directly follow the steps to the left to determine the LCD:* Factor(s) of first denominator: $\left(x-1\right), (x+1)$
* Factor(s) of second denominator: $(x-1)$
* Common factors: $(x-1)$
* Unique factors: $(x+1)$

Then the LCD is the product of the common and unique factors: $\left(x-1\right)(x+1)$ |
| Rewrite each rational expression as an equivalent expression with the LCD as the denominator. | $$\frac{2x}{\left(x-1\right)(x+1)}-\frac{4}{x-1}∙\frac{(x+1)}{(x+1)}, x\ne -1, 1$$$$=\frac{2x}{\left(x-1\right)(x+1)}-\frac{4(x+1)}{\left(x-1\right)(x+1)}$$ |
| Determine the difference of the numerators of the equivalent expressions. | $$=\frac{2x-4(x+1)}{\left(x-1\right)(x+1)}=\frac{2x-4x-4}{\left(x-1\right)(x+1)}=\frac{-2x-4}{\left(x-1\right)(x+1)}$$ |
| Simplify the rational expression and restate the restrictions on the variable. The non-permissible values of the variable in the simplified expression are the combination of the non-permissible values of the original expression. | $$\frac{-2(x+2)}{\left(x-1\right)(x+1)}, x\ne -1, 1$$**To ensure that the final expression is fully simplified, double-check that there are no common factors in the numerator and denominator. This particular expression is fully simplified.** |

**4.5 Solving Rational Equations** (pp. 253 – 261)

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| **Term** | **Definition / Equation** | **Example** |
| Rational Equation | An equation that involves one or more rational expressions. | Example of a rational equation:$$\frac{5}{x}=\frac{4}{x+2}$$ |
| Extraneous Root | A solution to an equation that is **not permissible in the original equation.**If a root of a rational equation is a non-permissible value of the rational expressions in the original equation, it is an extraneous root and must be dismissed as a valid solution. | Algebraically solving $\frac{18}{x(x-3)}=\frac{6}{x-3}-\frac{5}{x}$ produces the root $x=3$. However, $x=3$ **is a non-permissible value for the original equation. Hence, it is an extraneous root, and the equation does not have a solution.** |
| Inadmissible Solution | Solution that is permissible in the original equation but **is not valid in the context of the problem.** | See Example 3 on p. 255. |

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| **Example 1. Solving a rational equation.**Solve:$$\frac{x}{2}-\frac{2x+5}{4}=\frac{4}{3}$$ |
| **Strategy / Step** | **Solution** |
| Factor rational expressions to determine any non-permissible values and to determine the lowest common denominator (LCD). State restrictions on the variable. | **None of the rational expressions in this equation can be factored further. There are no non-permissible values because there are no variables in the denominators.****The LCD is 12.** |
| Multiply each term in the equation by the lowest common denominator (LCD) to eliminate fractions from the equation. | $$12∙\frac{x}{2}-12∙\frac{2x+5}{4}=12∙\frac{4}{3}$$$$6x-3\left(2x+5\right)=16$$$$6x-6x+5=16$$ |
| Solve the resulting linear or quadratic equation. | $$5=16$$**But** $5\ne 16$**, so there is no solution.** |
| Check for extraneous roots always, and for inadmissible solutions in contextual problems. | **This equation has no solution.** |
| Verify the solution by substituting into the original equation to ensure that LHS=RHS. | **There is no solution to substitute.** |

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| **Example 2. Solving a rational equation.**Solve:$$\frac{2}{a+2}-\frac{a^{2}+4}{a^{2}-4}=\frac{a}{2-a}$$ |
| **Strategy / Step** | **Solution** |
| Factor rational expressions to determine any non-permissible values and to determine the lowest common denominator (LCD). State restrictions on the variable. | $$\frac{2}{a+2}-\frac{a^{2}+4}{\left(a+2\right)(a-2)}=\frac{-a}{a-2}$$**Restrictions on the variable:** $a\ne -2, 2$**LCD:** $\left(a+2\right)(a-2)$ |
| Multiply each term in the equation by the lowest common denominator (LCD) to eliminate fractions from the equation. | $$\left(a+2\right)\left(a-2\right)∙\frac{2}{a+2}-\left(a+2\right)\left(a-2\right)∙\frac{a^{2}+4}{\left(a+2\right)\left(a-2\right)}=\left(a+2\right)(a-2)∙\frac{-a}{a-2}$$$$2\left(a-2\right)-\left(a^{2}+4\right)=\left(a+2\right)(-a)$$$$2a-4-a^{2}-4=-a^{2}-2a$$ |
| Solve the resulting linear or quadratic equation. | $$-a^{2}+2a-8=-a^{2}-2a$$$$4a-8=0$$$$4a=8$$$$a=2$$ |
| Check for extraneous roots always, and for inadmissible solutions in contextual problems. | $a=2$ **is an extraneous root since we already know that** $a\ne 2$**. This equation has no solution.** |
| Verify the solution by substituting into the original equation to ensure that LHS=RHS. | **There is no solution to substitute.** |

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| **Example 3. Solving a rational equation with an inadmissible solution.**See Example 3 on p. 255. |

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| **Example 4. Using a rational equation to model and solve a problem.**Rima bought a case of concert T-shirts for $450. She kept two T-shirts for herself and sold the rest for $560, making a profit of $10 on each T-shirt. How many T-shirts were in the case? |
| **Strategy / Step** | **Solution** |
| Define your variable and use the information in the problem to develop your equation. (Don’t get overwhelmed by all of the information. Read and re-read, slowly organizing the information in mathematical form.) | Let *x* = **number of T-shirts in the case**Buying price per T-shirt = $\frac{450}{x} $Selling price per T-shirt = $\frac{560}{x-2}$Profit per T-shirt = **Selling price per T-shirt – Buying price per T-shirt**Profit per T-shirt = $\frac{560}{x-2}-\frac{450}{x}$$$10=\frac{560}{x-2}-\frac{450}{x}$$ |
| Factor rational expressions to determine any non-permissible values and to determine the lowest common denominator (LCD). State restrictions on the variable. | The rational expressions are fully factored already.Non-permissible values: $x=0, 2$Considering inadmissible values: $x>2$ **since Rima kept 2 T-shirts, and still had some left to sell. Also,** $xϵW$ **since a fractional T-shirt cannot be sold.**→ Overall restrictions on the variable: $x>2, xϵW$LCD: $x(x-2)$ |
| Multiply each term in the equation by the lowest common denominator (LCD) to eliminate fractions from the equation. | $$10∙x(x-2)=\frac{560}{x-2}∙x(x-2)-\frac{450}{x}∙x(x-2)$$$$10x\left(x-2\right)=560x-450(x-2)$$ |
| Solve the resulting linear or quadratic equation. | $$10x^{2}-20x=560x-450x+900$$$$10x^{2}-20x=110x+900$$$$10x^{2}-130x-900=0$$$$10\left(x^{2}-13-90\right)=0$$$$10\left(x-18\right)\left(x+5\right)=0$$$$x=18,-5$$ |
| Check for extraneous roots always, and for inadmissible solutions in contextual problems. | $x=-5$ **is an inadmissible solution since it does not make sense for a case to hold a negative number of T-shirts. Thus, the only solution that works is** $x=18$**.** |
| Verify the solution by substituting into the original equation to ensure that LHS=RHS. | **Substituting** $x=18$ **into the original equation yields LHS=RHS.****There were 18 T-shirts in the case.** |

**4 Chapter Self-Test** (p. 262)

**4 Chapter Review** (pp. 263 – 266)

Q1) Why is factoring important when you simplify a rational expression.

A1) Factoring can help you:

 **- determine non-permissible values.**

 **- simplify a rational expression or the product or quotient of rational expressions.**

 **- determine the lowest common denominator (LCD) when you are adding or subtracting rational expressions.**

Q2) How do you solve and verify the roots of a rational equation?

A2) **You can solve a rational equation algebraically by multiplying each term in the equation by the LCD and then solving the resulting polynomial equation. You can verify the roots by substituting them into the original equation to ensure that the left-hand side equals the right-hand side.**

**Math 30-2 Notes**

**Chapter 5: Polynomial Functions**

**(pp. 271 – 329)**

**5.1 Exploring the Graphs of Polynomial Functions**  (pp. 274 – 277)

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| **Term** | **Definition / Equation** | **Example** |
| End Behaviour | The description of the shape of the graph, from **left to right, on the coordinate plane.** | For a polynomial of degree 1, the end behaviour is described as: Line extends either from quadrant II to quadrant I, or from quadrant II to quadrant IV.Use the grid below to sketch the two possibilities. |
| Cartesian Grid | A grid divided into four quadrants by the x-axis and the y-axis. Quadrants are identified using roman numerals from I to IV, starting from the top right and progressing counter-clockwise around the origin. |  |
| Cubic Function | A polynomial function of the **third degree, whose greatest exponent is three.** | $$f\left(x\right)=7x^{3}-x^{2}-3x+2$$ |
| Turning Point | Any point where the graph of a function changes from **increasing to decreasing or from decreasing to increasing.** |

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| Graph with **two turning points, since the y-values change from decreasing to increasing, then from increasing to decreasing:** | Graph with **no turning points, since the y-values are always decreasing:** |

 |
| Polynomial Function in One Variable | A function that contains only the operations of multiplication and addition, with real-number coefficients, **whole-number exponents, and two variables.** The degree of the function is the **greatest exponent of the function.** The degree determines the **shape of the function.** | $f\left(x\right)=2x^{3}-x^{2}+5x+7$ is a **cubic polynomial function of degree 3.**$$f\left(x\right)=3$$is a **constant polynomial function of degree 0.**$$f\left(x\right)=2x-5$$is a **linear polynomial function of degree 1.** |
| Polynomial Functions Named According to Degree | Constant function, degree **0** | $f\left(x\right)=3x^{0}$ **or** $f\left(x\right)=3$ |
| Linear function, degree **1** | $f\left(x\right)=5x^{1}+1$ **or** $f\left(x\right)=5x+1$ |
| Quadratic function, degree **2** | $$f\left(x\right)=x^{2}-2x+1$$ |
| Cubic function, degree **3** | $$f\left(x\right)=2x^{3}-5x+3$$ |
| Note: Terms are typically written so that the powers are in **descending order.** |

* Graphs of polynomial functions of the same degree have common characteristics.
* Sample sketches of functions, and all possibilities for the x-intercepts, y-intercepts, end behaviour, range, and number of turning points for each function type are shown:



Image source: *Principles of Mathematics 12*, p. 276.

**5.2 Characteristics of the Equations of Polynomial Functions** (pp. 278 – 292)

|  |  |  |
| --- | --- | --- |
| **Term** | **Definition / Equation** | **Example** |
| Standard Form | For linear function: $f\left(x\right)=ax+b$ **where** $a\ne 0$ |  |
| For quadratic function:$f\left(x\right)=ax^{2}+bx+c$ **where** $a\ne 0$ |  |
| For cubic function:$f\left(x\right)=ax^{3}+bx^{2}+cx+d$ **where** $a\ne 0$ |  |
| Constant Term | The term that **does not have a variable.** | In this function, the constant term is **-3:**$$f\left(x\right)=4x^{3}+x-3$$ |
| Leading Coefficient | The coefficient of the term with the **greatest degree in a polynomial function.** | In this function, the leading coefficient is **4:**$$f\left(x\right)=4x^{3}+x-3$$ |

**Example 1.** Complete the following tables for the given polynomial functions.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Standard Form  | a) $f\left(x\right)=\frac{1}{2}x-6$ | b) $f\left(x\right)=-5x-2$ | c) $f\left(x\right)=-2x^{2}+2x+4$ | d) $f\left(x\right)=x^{2}-6x+12$ |
| Graph |  |  |  |  |
| Degree |  |  |  |  |
| Number ofx-intercepts |  |  |  |  |
| y-intercept |  |  |  |  |
| End Behaviour |  |  |  |  |
| Domain |  |  |  |  |
| Range |  |  |  |  |
| Number of Turning Points |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Standard Form  | a) $f\left(x\right)=-2x^{3}+4x^{2}- 3x+1$ | b) $f\left(x\right)=2x^{3}+4x^{2}- 3x+1$ | c) $f\left(x\right)=x^{3}-2x^{2}-15x+36$ | d) $f\left(x\right)=x^{3}-8$ |
| Graph |  |  |  |  |
| Degree |  |  |  |  |
| Number ofx-intercepts |  |  |  |  |
| y-intercept |  |  |  |  |
| End Behaviour |  |  |  |  |
| Domain |  |  |  |  |
| Range |  |  |  |  |
| Number of Turning Points |  |  |  |  |

**Apply the Math.** Work through Examples 1 – 3 on pp. 279 – 285 of the text.

**Key Ideas**

When a polynomial function is in standard form:

|  |  |
| --- | --- |
| 1. The maximum number of x-intercepts the graph may have is equal to…
 | **...the degree of the function.** |
| 1. The maximum number of turning points a graph may have is equal to…
 | **...one less than the degree of the function.** |
| 1. The degree and leading coefficient of the equation of a polynomial function indicate…
 | **…the end behaviour of the graph of the function.** |
| 1. The constant term in the equation of a polynomial function is…
 | **…the y-intercept of the graph.** |

|  |  |  |
| --- | --- | --- |
| **Polynomial Name** | **End Behaviour** | **Graph** |
| Linear & Cubic Polynomial Functions | If the leading coefficient is negative, then the graph of the function extends from **quadrant II to quadrant IV.** |  |
| If the leading coefficient is positive, then the graph of the function extends from **quadrant III to quadrant I.** |  |
| Quadratic Polynomial Functions | If the leading coefficient is negative, then the graph of the function extends from **quadrant III to quadrant IV.** |  |
| If the leading coefficient is positive, then the graph of the function extends from **quadrant II to quadrant I.** |  |

**5 Mid-Chapter Review** (pp. 293 – 294)

Q1) How can you tell whether a given equation represents a polynomial function?

A1) **A polynomial function is a function in which the coefficients are real numbers and the exponents of the variables are whole numbers. It involves only multiplication and/or addition of real numbers and variables.**

Q2) How can you describe the characteristics of the graph of a polynomial function by looking at the equation of the function?

A2) For a polynomial fuction:

* Maximum number of x-intercepts = **degree.**
* Constant term = **y-intercept.**
* For linear or cubic functions, if the leading coefficient is
	+ negative, the function extends from **quadrant** **II to quadrant IV.**
	+ positive, the function extends from **quadrant III to quadrant I.**
* For quadratic functions, if the leading coefficient is
	+ negative, the function extends from **quadrant III to quadrant IV.**
	+ positive, the function extends from **quadrant II to quadrant I.**
* Domain is $\left\{xϵR\right\}$**.**
* For linear and cubic functions, the range is $\left\{yϵR\right\}$.
* Quadratic functions have a range restricted by their max or min values: $\left\{y\geq minimum, yϵR\right\}$ **or** $\left\{y\leq maximum, yϵR\right\}$**.**
* If the function is
	+ cubic, there **are zero or two turning points.**
	+ quadratic, there **is one turning point that is either a max or min.**
	+ linear, there **is no turning point; the function is a line.**

**5.3 Modelling Data with a Line of Best Fit**  (pp. 295 – 306)

**Table 5.3.1.** Data to be used in the Examples for section 5.3. (Data source: *Principles of Mathematics 12*, p. 295.)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Height (cm) | 165.0 | 172.5 | 172.5 | 153.8 | 157.5 | 170.0 | 168.8 | 177.5 | 182.5 | 172.5 | 180.0 | 177.5 | 165.0 | 165.0 | 175.0 |
| Hand Span (cm) | 20.0 | 21.1 | 17.6 | 16.5 | 17.5 | 19.0 | 20.8 | 22.5 | 25.0 | 23.0 | 20.2 | 21.1 | 20.7 | 16.0 | 21.2 |

|  |  |  |
| --- | --- | --- |
| **Term** | **Definition / Equation** | **Example** |
| Independent Variable | The variable that is **being manipulated, and that is always placed on the horizontal axis of a graph.** | In Table 5.3.1, the independent variable is **height, since we are investigating how height affects hand span.** To input the data in the TI-83+:[Stat]→[Enter] to select Edit→position cursor on L1→[Clear]→[Enter] to clear any data in L1→move the cursor beneath L1→input height (independent variable) data with [Enter] between each value |
| Dependent VariableDependent Variable (cont’d) | The variable that is **being observed, and that is placed on the vertical axis of the graph.** | In Table 5.3.1, the dependent variable is **hand span, since we are investigating how hand span is affected by height.**[Stat]→[Enter] to select Edit→position cursor on L2→[Clear]→[Enter] to clear any data in L2→move the cursor beneath L2→input hand span (dependent variable) data with [Enter] between each value |
| Scatter Plot | A graph of plotted points that show the relationship between two quantities or phenomena (variables). Useful in showing the extent of correlation, if any, between the variables. | To create a scatter plot of the data, input the data in L1 and L2 using the methods described above. Then:[2nd][Y=] to select Stat Plot→[Enter] to select 1:Plot1→select: On, Scatter Plot Type (1st in Type list), Xlist should be L1, Ylist should be L2→[Zoom]→[9] to select 9:ZoomStat (Scatter plot should now be visible.) |
| Line of Best Fit | A straight line that best approximates the trend in a scatter plot.If the points on a scatter plot follow a linear trend, technology can be used, through linear regression, to determine the line of best fit.The equation of the line of best fit balances the points in the scatter plot on both sides of the line.Predictions can be made about values that are not recorded or plotted by **reading values from the line of best fit or using the equation of the line of best fit.** | To determine the line of best fit, first complete all of the steps above. Then, if the scatter plot seems to represent a linear relationship, you can determine the line of best fit:[Stat]→right cursor to move to Calc menu→[4] to select 4:LinReg (ax+b)→[Vars]→right cursor to move to Y-Vars menu→[1] to select 1:Function→[1] to select Y1→[Enter] (The values for *a*=slope and *b*=y-intercept should be displayed.)→[Graph] to see the line of best fit along with the scatter plot.For the given data, the equation for the line of best fit is:$$y=0.22573…x-18.30495…$$**where** $y=hand span$ **in cm and**$ x=height$ **in cm** |
| Regression Function | A line or curve of best fit, developed through a statistical analysis of the data.  | The line of best fit is a specific regression function. However, if our data did not appear to be linear, we would choose a different regression function. For example, we may choose 5:QuadReg instead of 4:LinReg if our data appeared to represent a quadratic relationship. |
| InterpolationInterpolation (cont’d) | The process used to estimate a value **within the domain of a set of data, based on a trend.** | To use interpolation to estimate a person’s hand span if they were 181 cm tall, we could **substitute *x*=181 cm into the equation for the line of best fit:**$$y=\left(0.22573…\right)\left(181\right)-18.30495…=22.55299…$$If you have completed all of the previous steps, you could have done the following:[2nd][Trace] to view the Calc menu→[1] to select 1:value→[1][8][1] to input the height→[Enter]→The y-value is displayed in the bottom right corner.**The estimate for a person’s hand span is about 22.6 cm if their height is 181 cm.** |
| Extrapolation | The process used to estimate a value **outside the domain of a set of data, based on a trend.** | To use extrapolation to estimate a person’s hand span if they were 150 cm tall, we could substitute ***x*=150 cm into the equation for the line of best fit:**$$y=\left(0.22573…\right)\left(150\right)-18.30495…=15.5552…$$If you have completed all of the previous steps, you could have done the following:[Window]→adjust the window so that Xmin is less than 150→[2nd][Trace] to view the Calc menu→[1] to select 1:value→[1][5][0] to input the height→[Enter]→The y-value is displayed in the bottom right corner.**The estimate for a person’s hand span is about 15.6 cm if their height is 150 cm.** |

**5.4 Modelling Data with a Curve of Best Fit** (pp. 307 – 317)

**Table 5.4.1.** Data to be used in the Examples for section 5.4. (Data source: *Principles of Mathematics 12*, pp. 310 – 311).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Years after 1979** | **Price of Gas (¢/L)** | **Years after 1979** | **Price of Gas (¢/L)** | **Years after 1979** | **Price of Gas (¢/L)** |
| 0 | 21.98 | 12 | 57.05 | 26 | 92.82 |
| 1 | 26.18 | 14 | 54.18 | 27 | 97.86 |
| 2 | 35.63 | 17 | 58.52 | 28 | 102.27 |
| 3 | 43.26 | 20 | 59.43 | 29 | 115.29 |
| 4 | 45.92 | 22 | 70.56 | 5 | 69.4 |
| 7 | 45.78 | 23 | 70.00 | 10 | 72.1 |
| 8 | 47.95 | 24 | 74.48 | 16 | 80.1 |
| 9 | 47.53 | 25 | 82.32 |  |  |

|  |  |  |
| --- | --- | --- |
| **Term** | **Definition / Equation** | **Example** |
| Independent Variable | The variable that is being **manipulated, and that is always placed on the horizontal axis of a graph.** | In Table 5.4.1, the independent variable is **Years after 1979, since we are investigating how the passage of time affects gas prices.** **To input the data in the TI-83+:****[Stat]→[Enter] to select Edit→position cursor on L1→[Clear]→[Enter] to clear any data in L1→move the cursor beneath L1→input Years after 1979 (independent variable) data with [Enter] between each value** |
| Dependent Variable | The variable that is being **observed, and that is placed on the vertical axis of the graph.** | In Table 5.4.1, the dependent variable is **Price of Gas, since we are observing how the price of gas changes over time.**[Stat]→[Enter] to select Edit→position cursor on L2→[Clear]→[Enter] to clear any data in L2→move the cursor beneath L2→input Price of Gas (dependent variable) data with [Enter] between each value |
| Scatter Plot | A graph of plotted points that show the relationship between two quantities or phenomena (variables). Useful in showing the extent of correlation, if any, between the variables. | To create a scatter plot of the data, input the data in L1 and L2 using the methods described above. Then:**[2nd][Y=] to select Stat Plot→[Enter] to select 1:Plot1→select: On, Scatter Plot Type (1st in Type list), Xlist should be L1, Ylist should be L2→[Zoom]→[9] to select 9:ZoomStat (Scatter plot should now be visible.)** |
| Curve of Best FitCurve of Best Fit (cont’d) | A curve that **best approximates the trend on a scatter plot.**If the points on a scatter plot seem to follow a predictable curved pattern, there may be a **quadratic or cubic relationship between the independent variable and the dependent variable.**Technology uses polynomial regression to determine the curve of best fit, resulting in an equation of a curve that **balances the points on both sides of the curve.**Predictions can be made by **reading values from the curve of best fit on a scatter plot or by using the equation of the curve of best fit.** | To determine the curve of best fit, first complete all of the steps above. Then, assess what type of function the data seems to approximate. This data approximates **a cubic relationship, so:**[Stat]→right cursor to move to Calc menu→[6] to select **6:CubicReg** →[Vars]→right cursor to move to Y-Vars menu→[1] to select 1:Function→[1] to select Y1→[Enter] (The values for *a*, *b*, *c* and *d* should be displayed.)→[Graph] to see the curve of best fit along with the scatter plot.For the given data, the equation for the curve of best fit is:$$y=0.01463…x^{3}-0.59916…x^{2}+8.25475…x+22.04851…$$**where** $y=Years since 1979$ **and**$x=Price of Gas$ **in ¢/L** |
| Interpolation | The process used to estimate a value **within the domain of a set of data, based on a trend.**To solve an equation, you can **use a system of equations method. Graph the corresponding function of each side of the equation. Ensure that your window is set such that the intersection(s) is(are) visible. The x-coordinate of the point of intersection is the solution to the equation.**  | Use interpolation to estimate the average price of gas in 1985. In this case, we would have $x=1985-1979=6$**. Wherever you see *x* in the equation for the curve of best fit, substitute in *x*=6.** If you have completed all of the previous steps, you could do the following:[2nd][Trace] to view the Calc menu→[1] to select 1:value→[6] to input the value for *x*→[Enter]→The y-value is displayed in the bottom right corner.The estimate for the price of gas in 1985 is **about 53.17¢/L.** |
| ExtrapolationExtrapolation (cont’d) | The process used to estimate a value **outside the domain of a set of data, based on a trend.**To solve an equation, you can **use a system of equations method. Graph the corresponding function of each side of the equation. Ensure that your window is set such that the intersection(s) is(are) visible. The x-coordinate of the point of intersection is the solution to the equation.**  | Use extrapolation to estimate the year in which the average price of gas will reach 200¢/L.In this case, we are given the y-value, and we want to estimate the x-value. After completing all of the previous steps:[Y=]→down cursor until beside Y2=→[2][0][0] to input the function $y=200$ representing the cost of gas→ [Graph] to determine if the window is set appropriately (should be able to see horizontal line at $y=200$ intersecting the cubic function→[Window]→adjust the settings to make the intersection visible→[2nd][Trace] to view the Calc menu→[5] to select 5:intersect→[Enter]→[Enter]→ [Enter]→The x-coordinate at the point of intersection is displayed in the lower left-hand corner as $x=34.77404….$ This represents **the number of years after 1979.** Thus, the year in which we estimate that the price of gas will reach 200¢/L is **2014 (since** $1979+35=2014)$**.** Yikes! How accurate do you think that this prediction really is? |

Some extra hints:

* Keep track of what your *x*-coordinates and your *y*-coordinates represent.
* Always double-check that you have input the data correctly.
* When interpolating or extrapolating, make sure that you understand clearly what variable you are predicting.

**5 Chapter Self-Test** (p. 318)

**5 Chapter Review**  (pp. 319 – 322)

Q1) Why is it better to use the regression function when interpolating from a set of data, rather than just using the data?

A1) **If the data does not have much scatter, then just reading the points may provide an accurate estimate. However, most real-life data has some scatter, in which case you should use regression for the purposes of interpolation. Regression, a statistical analysis, produces an equation that minimizes the error in your estimate. You can use the regression equation or line/curve of best fit to interpolate or extrapolate data values.**

Q2) How can you solve a problem involving a data set that can be modelled using a polynomial function?

A2) **You can use a spreadsheet program or the statistical functions on a calculator to perform a linear, quadratic, or cubic regression to determine the line or curve of best fit and the equation of the regression function that represents the relationship between the given data. You can interpolate or extrapolate values by tracing along the graph or by substituting values into the equation of the regression function.**

**4-5 Cumulative Review** (pp. 327 – 329)